# The Great Divergence with Frictional Labor Markets *

Bessy Liao

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**Abstract**

Since the 1980s, US cities with a growing share of high-skill workers have experienced higher growth in wages and rent than cities with a declining share of high-skill workers. I document novel empirical facts about this "Great Divergence" showing that high-skill, high-rent cities also experience a reduction in long-run unemployment rates. Since wage and unemployment rates are jointly determined, incorporating geographic variation in unemployment rates is quintessential in understanding the welfare implication of this divergence. This paper develops a spatial equilibrium model with frictional labor markets that give rise to unemployment, featuring workers of different skill levels that share a housing market. I calibrate the model to the US economy between 2005 and 2019 and find that the worker population is inefficiently small in high-wage, high-rent locations. The share of high-skill workers in these locations is inefficiently high. This misallocation is caused by the distortion resulting from the inseparability between the labor market and housing market location. Comparing the model to its competitive counterpart without unemployment shows that search frictions moderate the divergence, allowing an additional channel to balance the spatial equilibrium, leading to smaller utility differences between high- and low-skill workers. Policies that encourage low-skill workers to relocate to high-wage locations improve aggregate welfare.

**JEL Classification:** E24, J24, J31, J64, R13

**Keywords**: Unemployment Rate, Regional Divergence, Job Search, Skill Based, Labor Market Conditions, Spatial Equilibrium, Inequality

*University of California, Irvine. 3151 Social Science Plaza, Irvine, CA 92697; liaob2@uci.edu. This paper was previously circulated under "Trapped in Local Labor Markets". I thank my advisory committee members Jan K. Brueckner, Victor Ortego-Marti, Guillaume Rocheteau, and Miguel Zerecero for their guidance and invaluable support. I appreciate comments and discussion with Michael Choi, Fabian Eckert, Juliette Fournier, Paul Jackson, Miroslav Gabrovski, Eugenia Gonzalaz-Aguado, Ioannis Kospentaris, Cristina Lafuente Martinez, Jack Liebersohn, Enrico Moretti, Vegard Nygaard, Giovanni Peri, Todd Schoellman, Chris Tonetti, Guanyi Yang, Shengxing Zhang as well as seminar participants at UCR, UCI macro brown bag, Midwest Macro Fall 2022 meeting, WEAI International 2023 meeting, and the Urban Economics Association 2021 meeting.

# 1 Introduction

Divergence in wages between high-skill and low-skill workers has been well documented since the 1980s, most notably in works by Katz and Murphy [1992] and Goldin and Katz [2008]. Recent research has drawn attention to the spatial dimension of wage divergence, whereby Ganong and Shoag [2017], Hsieh and Moretti [2019] and Austin et al. [2018] document the wage divergence in terms of geography. Together, these trends in labor income reinforce each other, leading to a polarization of cities described by Moretti [2012] as "the Great Divergence", where an abundance of high-skill workers are clustered in high-wage and high-rent cities and the low-wage and low-rent cities bear a larger share of low-skill workers. Since wages are only observed conditional on employment, what matters for workers is the product of wages and employment probabilities. Therefore, understanding the welfare implications of the Great Divergence necessitates incorporating the spatial variation of unemployment rates caused by search frictions in the labor markets.

How do search frictions in the labor markets contribute to the Great Divergence? In this paper, I document the dispersion of unemployment rates in the US in terms of geography by skill level. Between 2005 and 2019, there was considerable geographic variation in unemployment rates, particularly for low-skill workers. Furthermore, the cities that have grown in high-skill concentration and real wages also experienced decreased unemployment rates for both skill types. I then develop and calibrate a spatial equilibrium model with heterogeneous workers in frictional labor markets and local housing markets to understand the implications of frictional labor markets on the location decisions of high- and low-skill workers. I ask how high-skill workers' location choices affect low-skill workers' location choices and vice versa. Using the calibrated model, I find the optimal skill composition of workers across space with search frictions in labor markets.

Locations fundamentally differ in their production function and housing supply in my model, which generates an equilibrium with two types of locations - locations with large shares of high-skill workers ($H$) feature high-wage, high-rent with low unemployment rates, and locations with small shares of high-skill workers ($L$) feature low-wage, low-rent with high unemployment rates. The negative association between wages and unemployment rates across locations results from the model's job creation condition since firms have incentives to create more jobs where the per-worker output is higher.

High-skill and low-skill workers affect one another through the following channels: First, due to the complementarity between high- and low-skill workers in the production process, locations with more high-skill workers pay higher wages for low-skill workers. This is the agglomeration force that creates incentives for high- and low-skill workers to co-locate. Second, due to the limited housing supply, high- and low-skill workers compete on the common housing market, raising the cost of living in high-wage locations, which is a dispersing force. The relative strength of these opposing forces determines the equilibrium size of labor markets as well as the skill composition in each of them.

I find that search frictions in the labor market moderate the divergence, resulting in high-wage, high-rent cities having a smaller share of high-skill workers compared to its competitive counterfactual. With search friction, the expected income gaps between high-wage, high-rent cities and low-wage, low-rent cities are narrowed, especially for low-skill workers. This is because high-wage, high-rent locations also feature much lower unemployment rates for low-skill workers, raising their expected wages and creating incentives for the low-skill workers to move there.

The decentralized equilibrium is never efficient, even when the Hosios [1990] efficiency condition is satisfied. The local housing markets distort workers' location choices, complicating the congestion externality and leading to workers' misallocation across locations. In random search models, inefficiency arises due to the missing price of market tightness. One way to implement the Hosios [1990] condition and to restore efficiency is charging an entry fee to workers such that the cost of participating in the labor market equals the cost of the congestion they create for other workers. In my model, however, the "entry fee" workers pay to participate is housing rent due to the inseparability of work and home location. Therefore, the entry fee price is not based on market tightness but is determined solely by a land clearing condition. Thus, even when the Hosios [1990] condition is imposed, the size of the de facto entry fee is distorted by the housing price. Therefore, market tightness is still mispriced by the housing rent, leading to the misallocation of workers across locations.

A calibrated version of the model with two representative locations, one high-skill intensive (location $H$) and one low-skill-intensive (location $L$), shows that search friction in the labor market lowers the share of high-skill workers in $H$ by 1.6%, reduces the real wage gap between locations by around 30% for both skill groups, and shrinks the location housing rent gap by 14%. Hence, search friction in the labor market moderates the Great Divergence. Comparing the outcomes of the planner's problem with the decentralized allocation, we can see that the labor force is inefficiently small in the more productive location, and the unemployment rate is inefficiently high for high-skill workers in both locations, whereas inefficiently low for low-skill workers in both locations. The constrained efficient allocation thus produces 5% more aggregate output than the decentralized allocation. Given the inefficiencies, I conduct a counterfactual experiment by giving low-skill workers relocation subsidies to encourage them to live in location $H$. The subsidy improves aggregate welfare and moves the equilibrium towards the constrained-efficient outcome.

The rest of the paper unfolds as follows. The remaining parts of the introduction present some motivating facts on the divergence of local labor markets and related literature. Section 2 presents a baseline model to illustrate intuition. Section 3 characterizes the equilibrium. Section 4 discusses how labor market search friction affects the Great Divergence, and Section 5 solves the planner's problem. Section 6 presents quantitative analyses. Lastly, Section 7 concludes.

## 1.1 Descriptive facts

This section presents some descriptive facts. I illustrate that between 2005-2019 (i) There are notable unemployment rates dispersions across metropolitan areas. The range of variation is much wider for low-skill workers than for high-skill workers; (ii) For both skill groups, growth in the share of the high-skill labor force is associated with the decrease in local unemployment rates; (iii) For both skill groups, the growth in wages is associated with the decrease in local unemployment rates.

Two definitions of high-skill workers are applied here. The first is defined through the worker's occupation; more detailed methodology can be found in Section 6.1. The worker's educational attainment defines the second one. A worker with a college degree or above is considered a high-skill worker and otherwise is a low-skill worker.

Figure 1 shows the variation of unemployment rates across locations. Panel (a) maps the high-skill workers' unemployment rates by MSAs and panel (b) maps low-skill workers' unemployment rates by MSAs. We can see that the 2005-2019 average MSA unemployment rate for high-skill workers ranges from 0.89% - 6.72%, whereas for low-skill workers, the range is much wider, ranging from 2%-13.98%.

Figure 2 shows that the growth of high-skill worker share is negatively associated with changes in unemployment rates. From panels (a) and (b), we can see that the relationship between the growth of the share of high-skill workers and the unemployment rate is stronger for low-skill workers than for high-skill workers.

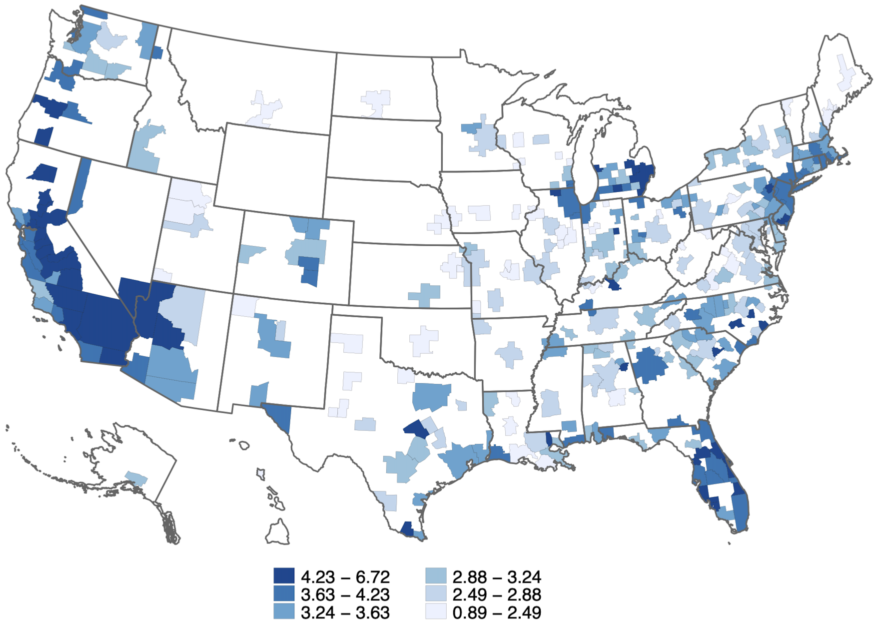
Figure 3 and Figure 4 show that the growth of nominal and real wages[1] are negatively associated with changes in unemployment rates. From panels (a) and (b) in both figures, we can see that similar to the spatial pattern presented in Figure 2, the relationship between the growth of real wages and changes in unemployment is stronger for low-skill workers than for high-skill workers. Together, Figure 2 - 4 show that unemployment rates have decreased in locations that have become more concentrated with high-skill workers and experienced growth in wages.

I ran one set of regressions to tease out the effect of high-skill share from the MSA fixed effects. Table 1 presents the effects of high-skill share on unemployment rates. Columns (1) and (2) show results for high-skill workers, whereas columns (3) and (4) show results for low-skill workers. Columns (1) and (3) use OLS regression to estimate the effect of the high-skill worker share on the unemployment rate, whereas column (2) and (4) uses local per capita patent counts as instruments for the share of high-skill workers. We can see from the negative coefficients that unemployment rates for both skill types are negatively correlated with the share of high-skill workers. The sizes of the coefficients are smaller for high-skill workers than for low-skill workers. Compared to IV regression outcomes in columns (2) and (4), the OLS regressions have a downward bias for both skill types. [2]Note that the regression outcomes are consistent with the graphical representations illustrated earlier in this section. Increases in the share of high-skill workers correlate with reduced unemployment rates for both skill types.

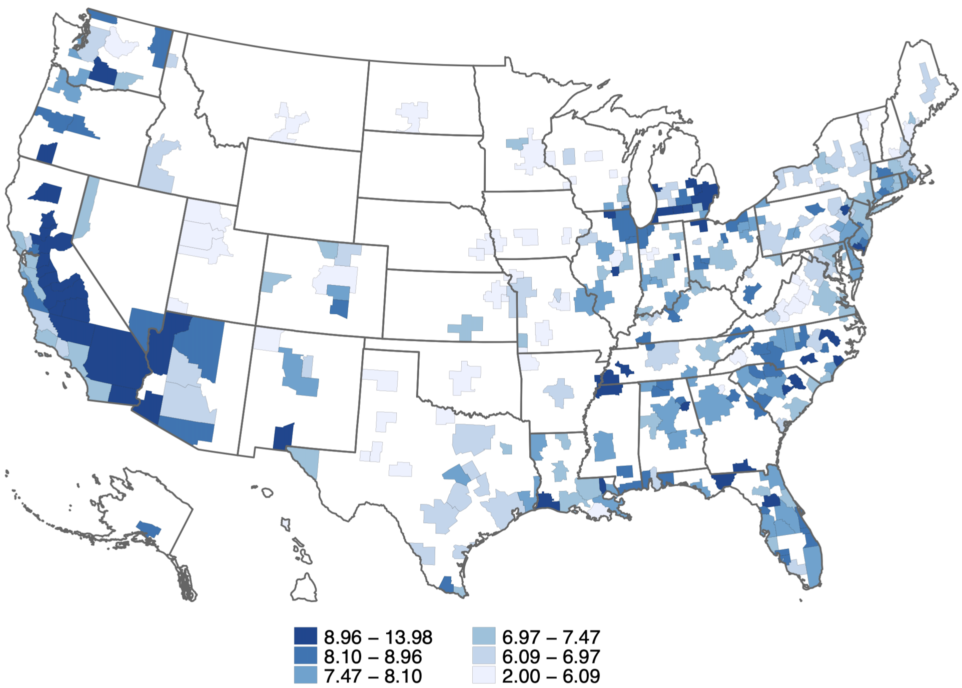
[1]Real wages calculated by discounting nominal wages by local housing prices.

[2]These two patterns persist for the education-based skill definition, referring to Table 11 in appendix A.1

Figure 1: Local unemployment rate by skills, 2005-2019 average




(a) High-skill workers

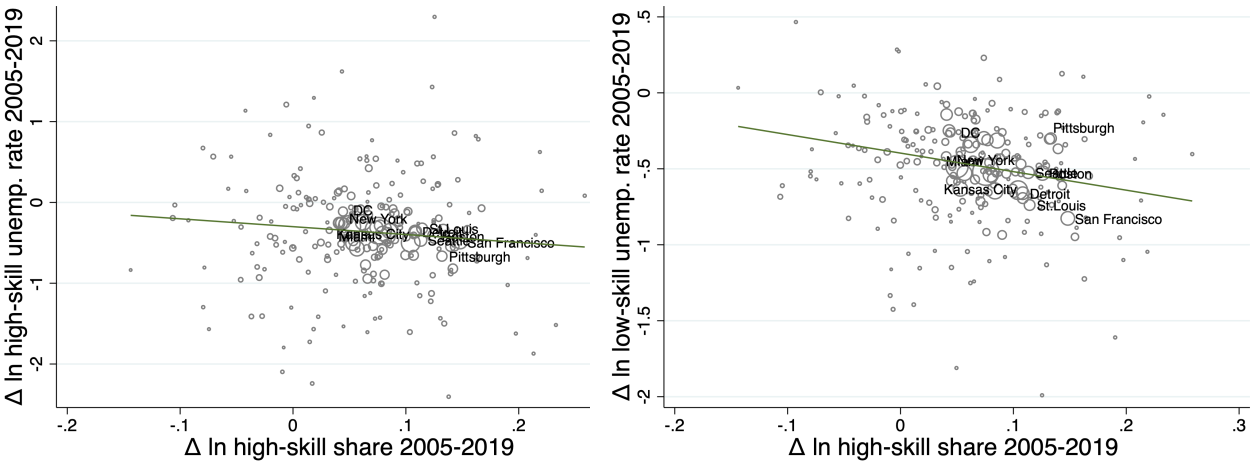



(b) Low-skill workers

*Notes: This map uses American Community Survey data from 2005-2019. Each block represents a metropolitan area (MSA). The skill definition used in the graph is occupation-based.*

Figure 2: Changes in Share of High-Skill Workers and Unemployment Rates by Skill Types, 2005-2019

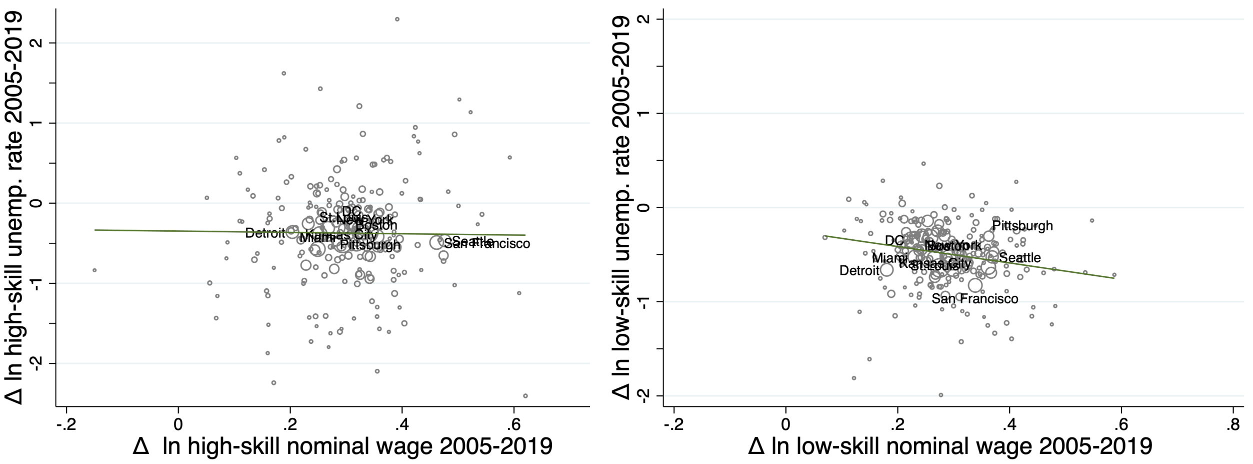
(a) High-skill workers (b) Low-skill workers




*Notes: This figure uses American Community Survey data from 2005-2019. Each circle represents a metropolitan area (MSA). The data points are weighted by the 2005 labor force size. The red line is the linear fit. The skill definition used in the graph is occupation-based. Graphs using the education-based skill definition can be found in appendix A.*

Figure 3: Changes in Nominal Wages and Unemployment Rate by Skill Types, 2005-2019

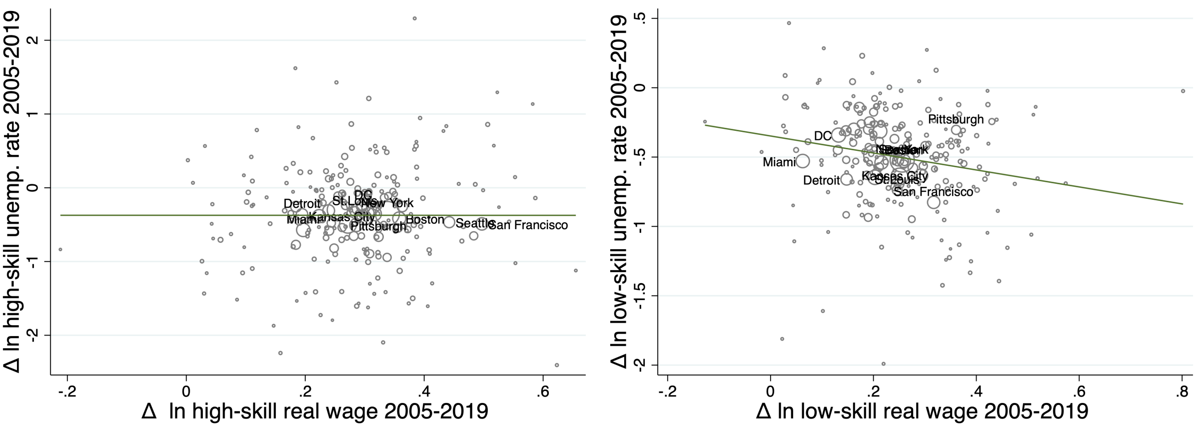
(a) High-skill workers (b) Low-skill workers




*Notes: This figure uses American Community Survey data from 2005-2019. Each circle represents a metropolitan area (MSA). The data points are weighted by the 2005 labor force size. The red line is the linear fit. The skill definition used in the graph is occupation-based. Graphs of education-based skill definition can be found in appendix A.*

Figure 4: Changes in Real Wages and Unemployment Rate by Skill Types, 2005-2019

(a) High-skill workers (b) Low-skill workers




*Notes: This figure uses American Community Survey data from 2005-2019. Each circle represents a metropolitan area (MSA). The data points are weighted by the 2005 labor force size. The red line is the linear fit. The skill definition used in the graph is occupation-based. Graphs of education-based skill definition can be found in appendix A.*

Table 1: Share of High-Skill Worker and Unemployment Rates

| | (1) | (2) | (3) | (4) |
|---|---|---|---|---|
| Unemployment Rate | High-Skill | High-Skill | Low -Skill | Low -Skill |
| | OLS | IV | OLS | IV |
| Log Share of High-Skill Worker (Occ) | -0.339*** | -0.0221 | -0.434*** | -0.227*** |
| | (0.0625) | (0.100) | (0.0369) | (0.0593) |
| Observations | 2,635 | 2,575 | 2,643 | 2,583 |
| R-squared | 0.321 | 0.316 | 0.407 | 0.406 |
| MSA FE | Yes | Yes | Yes | Yes |
| Year FE | Yes | Yes | Yes | Yes |

Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

## 1.2 Related Literature

This paper speaks to three threads of literature. First, this paper contributes to the body of work that studies the divergence between high-skill and low-skill workers regarding location choices. Shapiro [2006], Berry and Glaeser [2005], Moretti [2012], Eeckhout et al. [2021], Eckert et al. [2020] Giannone [2017] find that the critical driver of the spatial dispersion is due to productivity channels. They closely examine the sources of productivity changes in different cities, such as the concentration of college graduates, type of jobs, skill agglomeration, IT investment, skill- and information-intensive service industries, and skill-based technological changes. They find that productivity growth significantly affects wage premiums for high-skill workers but produces a much smaller effect for low-skill workers.

In addition to the productivity channels, Ganong and Shoag [2017], Gyourko et al. [2013] and Glaeser and Gyourko [2018] show that housing also plays a crucial role in the divergence of skill composition. They find that housing costs, housing price appreciation, and housing supply elasticity significantly contribute to the divergence of skill composition since, in highly productive locations, the prohibitively high housing prices crowd out lower-income households. In particular, the divergence is mainly explained by the highly inelastic land supply in the more attractive locations. These papers show that heterogeneity in productivity and housing supply matters for the divergence across locations. Thus, I incorporate such heterogeneity in a theoretical framework with frictional labor markets.

Second, this paper speaks to the literature on spatial differences in unemployment, where frictional local labor markets are studied in a geographic framework. This literature includes Kline and Moretti [2013], Kuhn et al. [2021], Bilal [2023] and Deschamps and Wilemme [2021]. These papers all study variations of the Diamond [1982] - Mortensen [1979] - Pissarides embedded in a Rosen [1979] - Roback [1982] spatial equilibrium. In particular, Kuhn et al. [2021] and Bilal [2023] emphasize the spatial differentials of job creation and job destruction. This paper focuses on the different spatial patterns of unemployment for high-skill and low-skill workers, a novel feature of this thread of literature. It examines their effects on the Great Divergence.

Lastly, this paper speaks to the body of work on spatial mismatch and optimal allocation of workers. Desmet and Rossi-Hansberg [2013] study the optimal city size, Fajgelbaum and Gaubert [2020] study the optimal allocation of workers across space. Acemoglu [2001] shows the skill composition of jobs can be inefficient when two types of jobs are created for one type of workers. Recent work by Hsieh and Moretti [2019] and Fournier [2020] shows that both interurban and intra-urban spatial misallocation leads to inefficiency. In contrast, Marinescu and Rathelot [2018] and Şahin et al. [2014] present evidence that geographical mismatch is present but is a minor driver in terms of the aggregate unemployment rate. This paper aims to enrich our understanding of the impact of spatial misallocation by bringing the heterogeneous skill levels and frictional labor market to the discussion. This has non-trivial welfare implications as the scale of misallocation can be masked by the heterogeneity of skill levels and employment status.

# 2 Environment

Time is continuous and indexed by $t \in \mathbb{R}_+$. There are $J$ locations. Each location $j \in \{1, ..., J\}$ is characterized by production technology and a housing supply $Q_j$.

### Production

Three types of goods are produced in each location: one final good $Z$ freely traded across locations and two intermediate goods $Y^s$,$Y^n$ produced by high-skill and low-skill workers, respectively. The final good $Z$ is treated as the numeraire and has a price of one. The production function of the final good is CES:

$$Z_j = \left[\sigma_j (Y_j^s)^\rho + (1-\sigma_j)(Y_j^n)^\rho\right]^{1/\rho} \ ,$$

where $\sigma_j$ is an exogenous parameter that indicates the relative importance of high-skill intermediate goods, and $1/(1-\rho)$ is the elasticity of substitution between the high-skill and low-skill input. The bigger the $\sigma_j$, the more important is the high-skill input. Therefore, if $\sigma_j > \sigma_k$, we say that location $j$ is high-skill-intensive and location $k$ is low-skill-intensive. The intermediate goods are non-storable and sold in competitive markets. There is a continuum of intermediate goods firms, and each chooses to produce either the $s$ or $n$ type of goods and hire one worker of that type. Intermediate goods productivity is denoted by $y^\chi$, where high-skill workers have higher intermediate goods productivity than low-skill workers, i.e., $y^s > y^n$. The total output of intermediate goods in location $j$ is equal to the sum of individual firms' production. This is equal to

$$Y_j^\chi = y^\chi (1 - u_j^\chi) L_j^\chi,$$

where $u_j^\chi$ denotes the unemployment rate for workers of skill type $\chi$ at location $j$, and $L_j^\chi$ is the labor force size of skill type $\chi$ at location $j$. Both $u_j^\chi$ and $L_j^\chi$ are determined by the equilibrium. Since the two intermediate goods are sold in competitive markets, their prices are equal to their marginal products in the production of the final good:

$$p_j^s = \sigma_j \left(Y_j^s\right)^{\rho-1} Z_j^{1-\rho}, \tag{1}$$

$$p_j^n = (1-\sigma_j) \left(Y_j^n\right)^{\rho-1} Z_j^{1-\rho}. \tag{2}$$

**Workers**

Workers are risk-neutral and discount the future at a rate $r > 0$. Their preferences over non-housing consumption $c_t$ and housing $h_t$ are

$$\mathbb{E} \int_0^\infty e^{-rt} \left(\frac{c_t}{1-\eta}\right)^{1-\eta} \left(\frac{h_t}{\eta}\right)^\eta dt. \tag{3}$$

There is a unit measure of workers of both skills. The total measure of high-skill workers is denoted by $\xi$, which is less than one, and the local labor force share of high-skill workers is $\zeta_j$, hence

$$\zeta_j = \frac{L_j^s}{L_j}, \tag{4}$$

$$1 = \sum_j L_j, \tag{5}$$

$$\xi = \sum_j \zeta_j L_j. \tag{6}$$

**Matching function**

The labor markets are segregated so that workers of skill type $\chi$ can only work for an intermediate firm of skill type $\chi$. For convenience, I use $\phi$ to denote the aggregate state of skill level and location $\phi = \{j \times \chi\}$. The matching function $m(\theta_\phi)$ between workers and intermediate goods firms depends on market tightness $\theta_\phi$ of each $\phi$, where $\theta_\phi \equiv \frac{v_\phi}{u_\phi}$. The matching function is Cobb-Douglas

$$m(u_\phi, v_\phi) = A u_\phi^\alpha v_\phi^{1-\alpha},$$

where $A$ is the matching parameter, $u_\phi$ is the unemployment rate for type $\phi$, and $v_\phi$ is the job vacancy rate. Since the matching function is homogeneous of degree of 1, the job finding rate $f(\theta_\phi)$ and vacancy filling rate $q(\theta_\phi)$ are

$$f(\theta_\phi) = \frac{m(u_\phi, v_\phi)}{u_\phi} = m(1, \theta_\phi); \quad q(\theta_\phi) = \frac{m(u_\phi, v_\phi)}{v_\phi} = m(1/\theta_\phi, 1). \tag{7}$$

Unemployed workers are free to move when unemployed and can only look for jobs where they live. Employed workers cannot move between locations but could quit their jobs and move.

**Housing clearing condition**

Each location has a housing supply $Q_j$, and absentee landowners own the land. They collect rents from workers in location j and use them to enjoy non-housing consumption $c_j^O$. In each location, $j$, total land supply equals total land demand. Therefore, the land-clearing condition for each location $j$ is

$$Q_j = \sum_\phi [h_\phi^b u_\phi + h_\phi(1 - u_\phi)] L_\phi \ , \tag{8}$$

where $h_\phi^b$ denotes housing consumption of unemployed worker of type $\phi$, and $h_\phi$ denotes housing consumption of employed worker of type $\phi$.

# 3 Equilibrium

The description of the equilibrium is presented as follows. I start by discussing the consumption decisions in Section 3.1, then I define the flow Bellman equations in Section 3.2 and Section 3.3 describes the spatial equilibrium. Section 3.4 discusses wage bargaining. Section 3.5 discusses the equilibrium conditions. Section 3.6 then defines a steady-state equilibrium. Section 3.7 discusses the implications of the equilibrium. And lastly, Section 3.8 and Section 3.9 discuss cross-skill interaction and present comparative statics.

## 3.1 Housing and non-housing consumption

All variables in the model are functions of $t$. To simplify notation, the $t$ argument is suppressed from now on. Since the utility function is Cobb-Douglas, the worker's consumption maximization problem would result in the share of spending on housing and non-housing consumption being fixed and governed by a parameter $\eta$. [3]Therefore, non-housing consumption $c_\phi^b$ for an unemployed worker and $c_\phi$ for an employed worker are

$$c_\phi^b = (1 - \eta) b^\chi, \quad c_\phi = (1 - \eta) w_\phi \ , \tag{9}$$

where $w_\phi$ is the wage, and $b^\chi$ is the unemployment benefit for workers of skill type $\chi$. Housing consumption $h_\phi^b$ for an unemployed worker and $h_\phi$ for an employed worker are

$$h_\phi^b = \frac{\eta b^\chi}{R_j}, \quad h_\phi = \frac{\eta w_\phi}{R_j}, \tag{10}$$

where $R_j$ is rent in location j, determined by plugging the housing consumption equation (10) into the housing clearing condition equation (8):

$$R_j = \frac{\eta L_j}{Q_j} \left[ [b^n u_j^n (1 - \zeta_j) + b^s u_j^s \zeta_j] + w_j^s (1 - u_j^s) \zeta_j + w_j^n (1 - u_j^n)(1 - \zeta_j) \right] . \tag{11}$$

[3]The derivation can be found in appendix B.1

Plugging the expression of optimal housing consumption, equation (9) and optimal non-housing consumption (10) into the utility function, equation (3), the indirect utility of an employed worker becomes $w_\phi R_j^{-\eta}$ and the indirect utility of an unemployed worker becomes $b^\chi R_j^{-\eta}$.

### 3.2 Bellman Equations

Let $U_\phi$, $W_\phi$, $V_\phi$, $J_\phi$ denote the value of the unemployed, the employed, an intermediate goods firm vacancy and a filled intermediate firm job for each $\phi$.[4] The Bellman equations involving these variables are:

$$rW_\phi = w_\phi R_j^{-\eta} + s^\chi(U_\phi - W_\phi), \tag{12}$$

$$rU_\phi = \max_j\{b^\chi R_j^{-\eta} + f(\theta_\phi)(W_\phi - U_\phi)\}, \tag{13}$$

$$rV_\phi = \max_j\{-k^\chi + q(\theta_\phi)(J_\phi - V_\phi)\}, \tag{14}$$

$$rJ_\phi = p_\phi y^\chi - w_\phi + s^\chi(V_\phi - J_\phi). \tag{15}$$

The first Bellman equation is an employed worker's flow value. The first term on the right-hand side is an employed worker's indirect utility, as discussed in Section 3.1. With probability $s^\chi$, a worker becomes unemployed and separated from her job. The second Bellman equation is an unemployed worker's flow value. Since an unemployed worker can move between locations, the worker chooses a location $j$ that maximizes utility. Like an employed worker, the first term on the right-hand side is the indirect utility of an unemployed worker. An unemployed worker meets a firm at rate $f(\theta_\phi)$.

The third Bellman equation is a vacant firm's flow value. Vacant firms are also free to choose where to locate, so they will choose location $j$ to maximize their profit. Once they settle in a location, they need to pay a vacancy cost $k^\chi$ that depends on the skill type. It is more costly to open a high-skill vacancy than a low-skill vacancy, i.e., $k^s > k^n$. A vacant firm meets an unemployed worker at rate $q(\theta_\phi)$. The last Bellman equation is the flow value of a filled firm. The firm's profit is the value of the output $p_\phi y^\chi$ less the wage paid to the worker. A match is exogenously destroyed at the rate $s$. The free entry condition of the firms implies $V_\phi = 0$; hence the max operator drops out of the equation (14). Using the last two Bellman equations, $J_\phi$ must satisfy both of the following:

$$J_\phi = \frac{k^\chi}{q(\theta_\phi)}; J_\phi = \frac{p_\phi y^\chi - w_\phi}{r + s^\chi}. \tag{16}$$

Rearranging equation (12) and equation (13) yields

$$(r + s^\chi)(W_\phi - U_\phi) = (w_\phi - b^\chi) R_j^{-\eta} - f(\theta_\phi)(W_\phi - U_\phi). \tag{17}$$

### 3.3 Spatial Equilibrium

Since unemployed workers are free to move between locations, if there are unemployed workers in different locations in equilibrium, they should be indifferent between the locations. Hence, their value will be the

[4] Note that only the intermediate goods firms need to match with workers in the frictional labor markets and the intermediate goods are sold in a competitive market in the production of final goods.

same for all locations, i.e., $U_j^\chi = U_{j'}^\chi = \overline{U}^\chi, \forall j, j' \in J$ where $\overline{U}^\chi$ denotes the common value for the unemployed worker of skill type $\chi$. Therefore, the max operator regarding location drops out of equation (13).

### 3.4 Wage Bargaining

Following Bilal [2023], I use an adjusted surplus, where the surplus for the worker is adjusted by the level of local rents so that the marginal utility of a dollar is equalized between the worker and firm since the worker's wage is discounted by it. The adjusted surplus is formulated as follows:

$$S_\phi = J_\phi + R_j^\eta [W_\phi - U_\phi].$$

Nash Bargaining determines the wage with an adjusted surplus, which yields

$$(1-\beta) R_j^\eta (W_\phi - U_\phi) = \beta (J_\phi - V_\phi), \tag{18}$$

where $\beta$ is worker bargaining power, and $V_\phi = 0$. Rearranging and plugging equation (16) into equation (18) yields

$$W_\phi - U_\phi = \frac{\beta}{(1-\beta) R_j^\eta} \frac{k^\chi}{q(\theta_\phi)}. \tag{19}$$

Plugging equation (17) into equation (19) yields

$$(1-\beta) R_j^\eta [(w_\phi - b^\chi) R_j^{-\eta} - f(\theta_\phi) \frac{\beta}{(1-\beta) R_j^\eta} \frac{k^\chi}{q(\theta_\phi)}] = \beta (p_\phi y^\chi - w_\phi). \tag{20}$$

Therefore, using equation (7) to eliminate $f(\theta_\phi)$ and $q(\theta_\phi)$, the expression for the wage is

$$w_\phi = \beta p_\phi y^\chi + [(1-\beta) b^\chi + \beta \theta_\phi k^\chi]. \tag{21}$$

### 3.5 Equilibrium Conditions

Plugging equation (16), (19) and (21) into equation (18), we have the job creation condition for each skill location group $\phi$:

$$\frac{k^\chi}{q(\theta_\phi)} = \frac{(1-\beta) p_\phi y^\chi - [(1-\beta) b^\chi + \beta \theta_\phi k^\chi]}{r + s^\chi}. \tag{22}$$

The left-hand side is the firm's expected cost of hiring a worker, where the location-specific vacancy cost is adjusted by the expected time to find a worker. The right-hand side is the firm's expected gain from opening the vacancy. The job creation condition thus shows that firms keep entering the market until the expected profit of a vacancy equals the expected cost.

**Spatial Equilibrium Condition**

The spatial equilibrium condition equates the value of an unemployed worker across locations. Unemployed workers of type $\chi$ enter location $j$ until their indirect utility is equalized across locations. Plugging expression of $(W_\phi - U_\phi)$, as in equation (19), into equation (13) yields the spatial equilibrium condition:

$$\overline{U}^\chi = \left( b^\chi + \frac{\beta}{1-\beta} k^\chi \theta_j^\chi \right) R_j^{-\eta}; \quad \forall j \in J. \tag{23}$$

The unemployed worker will choose their location based on market tightness and housing prices. Since equation (22) relates productivity ($p_\phi y^\chi$) and market tightness $\theta_\phi$, even though ($p_\phi y^\chi$) does not show up in the spatial equilibrium condition, it can be inferred the value of market tightness. The bigger the market tightness, the more likely they will be employed; the higher the rent, the more expensive it is to live there, and hence lower indirect utility. The unemployed workers will allocate themselves until this expression is equalized across locations.

**Beveridge Curve**

The Beveridge curve is given by the following:

$$u_\phi = \frac{s^\chi}{s^\chi + f(\theta_\phi)}. \tag{24}$$

## 3.6 Definition of equilibrium

**Definition 1.** *A steady-state equilibrium is* $\{w_\phi, u_\phi, \theta_\phi, p_\phi, \zeta_j, L_j, R_j\}$ *for* $\phi \in J \times \{s, n\}$ *and* $j \in J$ *such that: equations (1),(2), (5),(6),(11),(22),(23),(24) are satisfied for each* $\phi$ *and* $j$*.*

## 3.7 Equilibrium Properties

This section further explores the equilibrium conditions and the equilibrium properties.

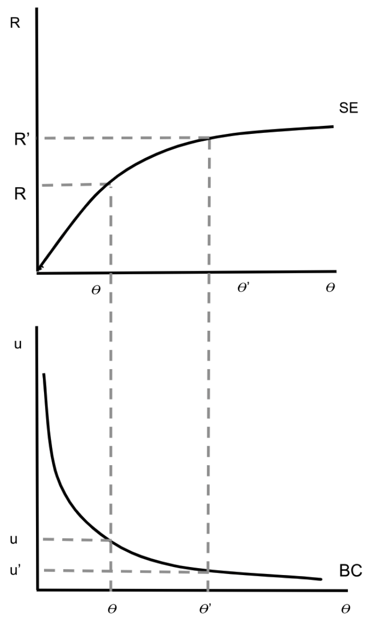



Figure 5: Spatial Equilibrium and Beveridge Curve

Plotting the spatial equilibrium condition in the top panel of Figure 5, we can see that within each skill type, workers are indifferent among points on an upward-sloping curve relating rent to market tightness. If workers choose location $j$ with a higher market tightness, then they are facing a higher $R_j^\eta$, so that if $\theta_j^\chi > \theta_{j'}^\chi$, then $R_j > R_{j'}$. Plotting the Beveridge Curve in the bottom panel of Figure 5, we can see that the unemployment rate is lower within the same skill group in locations with bigger market tightness. Therefore if $R_j > R_{j'}$, then $u_j^\chi < u_{j'}^\chi$.

**Proposition 1.** *Within each skill type, rent and market tightness are positively related across locations, while unemployment decreases with market tightness. Therefore, rent and unemployment rate are negatively related across locations, i.e., if $R_{j'} > R_j$, then $\theta_{j'}^\chi > \theta_j^\chi$, and $u_{j'}^\chi < u_j^\chi$.*

*Proof.* See Appendix F.1 □

Next, I examine the implication of the equilibrium on the relationship between the ranking of wages and unemployment rates across locations. For the job creation condition (22) to hold, an increase in the market tightness $\theta_\phi$ would raise the price of the intermediate goods $p_\phi$. Therefore, within each skill type $\chi$, if $\theta_j^\chi > \theta_k^\chi$, then $p_j^\chi > p_k^\chi$. By wage equation (21), we can see that $w_\phi$ increases with $p_\phi$ and $\theta_\phi$. Since we already know that $p_\phi$ also increases with $\theta_\phi$, we can say that $w_\phi$ increases with $\theta_\phi$, hence if $\theta_j^\chi > \theta_k^\chi$, then $w_j^\chi > w_k^\chi$. By the Beveridge Curve (24), we know that the unemployment rate is decreasing in market tightness. Therefore, we can see that workers receive higher wages for the same skill level in locations with lower unemployment rates: $u_j^\chi < u_{j'}^\chi$, then $w_j^\chi > w_{j'}^\chi$ $\forall j, j' \in J$ and $\chi \in \{s, n\}$.

**Corollary 1.** *Within each skill type, a location with a lower unemployment rate has a higher nominal wage. i.e. if $u_j^\chi < u_{j'}^\chi$, then $w_j^\chi > w_{j'}^\chi$. $\forall j, j' \in J$ and $\chi \in \{s, n\}$.*

*Proof.* See Appendix F.2 □

**Corollary 2.** *Within each skill type, a location with a lower unemployment rate has a higher real wage if its $\frac{p_j^\chi y^\chi}{R_j^\eta}$ is bigger. i.e. If $\frac{p_j^\chi y^\chi}{R_j^\eta} > \frac{p_{j'}^\chi y^\chi}{R_{j'}^\eta}$, then $u_j^\chi < u_{j'}^\chi$, then $\tilde{w}_j^\chi > \tilde{w}_{j'}^\chi$.*

*Proof.* See Appendix F.3 □

Corollary 1 establishes the relationship between the ranking of wages and the ranking of unemployment within each skill type across locations. It is critical to understand how the dispersion of unemployment rates shapes the great divergence. It shows that unemployment rates are lower within each skill group in locations where wages are higher, which maps the descriptive facts shown in Figure 3. Corollary 2 shows that the theoretical relationship between real wages ($\tilde{w}_j^\chi$) and unemployment rates is less conclusive, and the relationship depends on the size of the parameters. The intuition for Corollary 2 is that if the output difference dominates the rent difference between locations, then, for both types of workers, the location with higher real wages features lower unemployment rates. On the other hand, if the rent difference dominates the output difference between locations, then, for both types of workers, the location with higher real wages features lower unemployment rates. Section 6 presents the relationship between real wages and unemployment rates the calibrated model generated. It matches with Figure 3 presented in Section 1.1.

### 3.8 Comparative Statics

Considering the case of two locations $j \in \{H, L\}$, where $H$ is a high-skill-intensive location and $L$ is a low-skill-intensive location, with $\sigma_H > \sigma_L$. I am interested in understanding how location- and skill-based parameters affect the equilibrium outcomes, particularly their effects on market tightness, labor force sizes, high-skill share, and unemployment rates. Proposition 2 summarizes the comparative statics for location parameters, and Proposition 3 summarizes comparative statics for skill parameters.

**Proposition 2.** *Assuming skill dependent parameters are symmetrical, i.e.* $\xi = 0.5; y^s = y^n; b^s = b^n; k^s = k^n$*, comparative statics regarding the location parameters are summarized in Table 2.*

Table 2: Comparative Statics of Location Parameters

| | $\theta^s_j/\theta^n_j$ | $u^s_j/u^n_j$ | $L_j/L_{j'}$ | $\zeta_j/\zeta_{j'}$ |
|---|---|---|---|---|
| $Q_j/Q_{j'}\uparrow$ | $\rightarrow$ | $\rightarrow$ | $\uparrow$ | $\rightarrow$ |
| $\sigma_j/\sigma_{j'}\uparrow$ | $\uparrow$ | $\downarrow$ | $\uparrow$ | $\uparrow$ |

*Proof.* See Appendix F.4 □

As the housing supply in location $j$, $Q_j$, increases relative to $Q_{j'}$, the only thing in the equilibrium affected is housing rent $R_j$, which decreases with $Q_j$. Therefore, more workers of both types will move to location j; hence, $L_j$ increases relative to $L_{j'}$. Market tightness and unemployment rates are not affected, as indicated by the horizontal arrows.

As $\sigma_j$ increases relative to $\sigma_{j'}$, there is more demand for high-skill workers. Hence, firms will create more high-skill openings in location $j$ and location $j'$. More high-skill workers move to location j; hence, the unemployment rate is lower for high-skill workers, particularly those in location j. However, since there are fewer job openings for low-skill workers and the population share of low-skill workers is fixed, the unemployment rates are higher for low-skill workers in both locations. The relative labor force size is pinned down using spatial equilibrium conditions, which say that workers of both types are indifferent between locations when the disadvantages of costly rent balance the advantages of market tightness. Since location j has higher market tightness for both types, more workers will enter location j despite its higher rent. Therefore, location $j$'s worker size expands as $\sigma_j$ increases.

**Proposition 3.** *Assuming location-based parameters are equal, i.e.* $\sigma_j = \sigma_{j'} = 0.5$*,* $Q_j = Q_{j'}$*, comparative statics regarding the skill parameters are summarized in Table 3.*

Table 3: Comparative Statics of Skill Parameters

| | $\theta^s_j/\theta^n_j$ | $u^s_j/u^n_j$ | $L_j/L_{j'}$ | $\zeta_j/\zeta_{j'}$ |
|---|---|---|---|---|
| $y^s/y^n\uparrow$ | $\rightarrow$ | $\rightarrow$ | $\rightarrow$ | $\rightarrow$ |
| $b^s/b^n\uparrow$ | $\downarrow$ | $\uparrow$ | $\rightarrow$ | $\rightarrow$ |
| $k^s/k^n\uparrow$ | $\downarrow$ | $\uparrow$ | $\rightarrow$ | $\rightarrow$ |

*Proof.* See Appendix F.5 □

As $y^s$ increases relative to $y^n$, surplus for both types of matches increases at the same rate since $\sigma_H = \sigma_L = 0.5$. Therefore, market tightness increases for all skill-location groups, and unemployment decreases. Since the locations are symmetrical, there will be an equal number of workers in each location, and the share of high-skill workers is equalized between locations.

As $b^s$ increases relative to $b^n$, unemployment becomes more attractive for high-skill workers. Hence, the unemployment rate becomes higher for them. Therefore, market tightness decreases for high-skill workers. Due to the complementarity of high-skill output and low-skill output, being employed becomes less attractive for low-skill workers since fewer high-skill workers are employed. Therefore, the unemployment rate for low-skill workers also decreases, but since it is the second-order effect of the increase in $b^s$, the magnitude of the decreases is much smaller than for high-skill workers. Since the locations are symmetrical, there will be an equal number of workers in each location, and the share of high-skill workers is equalized between locations.

As $k^s$ increases relative to $k^n$, opening a high-skill vacancy becomes more expensive. Therefore, market tightness decreases, and unemployment increases for high-skill workers. Due to the complementarity of high-skill output and low-skill output, higher unemployment of high-skill workers means that employment becomes less attractive for low-skill workers. Therefore, the unemployment rate for low-skill workers also decreases, but since it is a second-order effect of the increase in $k^s$, the magnitude of the decrease is much smaller than for high-skill workers. Since the locations are symmetrical, there will be an equal number of workers in each location, and the share of high-skill workers is equalized between locations.

The comparative statics show that only the location-related parameters affect the skill composition and worker allocation across locations. In the absence of asymmetry of location-related parameters, skill-related parameters only affect differences in unemployment rates and market tightness between skill levels but do not affect the allocation of workers across locations.

## 3.9 Cross-skill Interaction

The labor markets are segregated by skill level. Therefore, the high-skill and low-skill workers will not be creating labor market congestions for workers of the other skill type. That is to say, a high-skill worker's decision to look for jobs in location $j$ does not make it less likely for a low-skill worker to find a job in location $j$, and vice versa. Nevertheless, high(low)-skill workers' decision to look for jobs in location $j$ can still affect the labor market outcome for low(high)-skill workers. The cross-skill interaction still occurs through two channels. First, it occurs through the production channel.

The final good is produced using both high-skill intermediate goods and low-skill intermediate goods, and they are complementary in final goods production. More high-skill workers employed in a location increases the marginal productivity of low-skill intermediate goods, which augments demand for low-skill workers. This is an attraction force that encourages high- and low-skill workers to co-locate.

On the other hand, high-skill workers and low-skill workers share a common housing market. Without an unlimited housing supply, they raise the housing cost for each other, limiting the size of the labor market in a location. This is the dispersing force. The relative strength of these two forces pins down the labor market sizes as well as the skill composition in each one of them.

# 4 Search Frictions and the Great Divergence

To understand whether frictions in the labor market exacerbate or alleviate the concentration of high-skill workers in highly productive locations, I compare the model with search friction with the models where the labor market is competitive[5]. In the competitive labor market, wages are

$$\breve{w}_j^s = \sigma_j(\breve{L}_j^s)^{\rho-1}(y^s)^\rho(y^n\breve{L}_j^n)^{1-\rho} = \breve{p}_j^s y^s, \tag{25}$$

$$\breve{w}_j^n = (1-\sigma_j)(\breve{L}_j^n)^{-\rho}(y^n)^{1-\rho}(y^s\breve{L}_j^s)^\rho = \breve{p}_j^n y^n. \tag{26}$$

Rent is

$$\breve{R}_j = \frac{\eta\breve{L}_j}{Q_j}[\breve{w}_j^s\zeta_j + \breve{w}_j^n(1-\zeta_j)]. \tag{27}$$

The labor clearing condition is the same as in the model with frictional labor markets

$$\sum_j \breve{L}_j = 1; \quad \sum_j \breve{L}_j\zeta_j = \xi. \tag{28}$$

The spatial equilibrium condition states that, within the same skill type, the worker's utility is the same in all locations

$$\breve{U}^\chi = \breve{w}_j^\chi \breve{R}_j^{-\eta}. \tag{29}$$

**Definition 2.** *A steady-state equilibrium with competitive labor markets is* $\{\breve{w}_\phi, \breve{\zeta}_j, \breve{L}_j, \breve{R}_j\}$ *for* $\phi \in J \times \{s, n\}$ *such that equations (25),(26),(27),(28),(33) are satisfied.*

In an economy without labor market search frictions, wages depend on the marginal output of the worker. Therefore, the wage gap between the locations will be

$$\Delta\breve{w}^\chi = (\breve{p}_j^\chi - \breve{p}_{j'}^\chi)y^\chi = \Delta\breve{p}^\chi y^\chi. \tag{30}$$

However, in an economy with labor market search frictions, wages depend on both the marginal output of the worker and the market tightness. Therefore, the wage gap between the locations depends on both the gap of marginal productivity and the gap between market tightness.

$$\Delta w^\chi = \beta y^\chi(p_j^\chi - p_{j'}^\chi) + \beta k^\chi(\theta_j^\chi - \theta_{j'}^\chi) = \beta y^\chi\Delta p^\chi + \beta k^\chi\Delta\theta^\chi. \tag{31}$$

Since $\beta < 1$, the contribution of marginal output in the wage gap is smaller than that in the competitive version. The differences in market tightness between locations also contribute to the wage gap.

Therefore, the wage gap between locations is bigger in the competitive labor market if $\Delta\breve{p}^\chi y^\chi > \beta y^\chi\Delta p^\chi + \beta k^\chi\Delta\theta^\chi$. And the wage gap between locations is smaller in the competitive labor market if $\Delta\breve{p}^\chi y^\chi < \beta y^\chi\Delta p^\chi + \beta k^\chi\Delta\theta^\chi$. It is summarized in Proposition 4.

[5]Note that the competitive labor market models where wages are the marginal product of labor are different from a model where matching efficiency reaches infinity, which still preserves the wage bargaining structure.

**Proposition 4.** *For both skill types, the wage gap between locations is bigger in the frictional labor market model if*

$$\Delta \breve{p}^{\chi} y^{\chi} - [\beta \Delta p^{\chi} y^{\chi} - \beta k^{\chi} \Delta \theta^{\chi}] > 0.$$

*Otherwise, the wage gap between locations is smaller in the frictional labor market model.*

*Proof.* See Appendix F.6. □

In the competitive labor market, the spatial equilibrium condition indicates that workers' location choices are based on the relative sizes of wages and housing prices. The relative size of the nominal wage and housing price pins down the spatial allocation of high- and low-skill workers. Nevertheless, in the benchmark economy with search frictions, the spatial equilibrium condition indicates that workers' location choices depend on the relative size of housing prices and market tightness, which affects both wages and unemployment rates. Re-arrange the job creation condition, and we can see that the spatial equilibrium condition for the economy with search friction is

$$\overline{U}^{\chi} = \left[ p_{\phi} y^{\chi} - \frac{r + s^{\chi}}{1 - \beta} \frac{k^{\chi}}{q(\theta_{\phi})} \right] R_j^{-\eta} \tag{32}$$

The spatial equilibrium for the economy without search friction is

$$\breve{\overline{U}}^{\chi} = [\breve{p}_{\phi} y^{\chi}] \, \breve{R}_j^{-\eta}. \tag{33}$$

since $\breve{w}_{\phi} = \breve{p}_{\phi} y^{\chi}$. We can see that in the frictional model, both productivity and market tightness play roles in determining the spatial equilibrium. However, the differences in market tightness are dampening the productivity differences between locations since the second term that contains that market tightness in equation (32) is subtracted from the first term in the equation. Section 6.4 quantitatively studies how the allocation of workers across space in labor markets with search friction differs from that in competitive labor markets.

# 5 Planner's problem

The social planner aims to maximize a social welfare function subject to a resource constraint and the law of motion of unemployment. The social welfare function assigns equal welfare weights for the three types of agents: high-skill workers, low-skill workers, and absentee landlords. Let $N_{\phi}$ denote the number of unemployed workers of each skill-location group $\phi$ and $E_{\phi}$ denote the number of employed workers.

The planner's objective function is

$$\omega = \int_0^{\infty} e^{-rt} \left( \sum_{\phi} \left[ \left( \frac{c_{\phi}^E}{1-\eta} \right)^{1-\eta} \left( \frac{h_{\phi}^E}{\eta} \right)^{\eta} \times E_{\phi} + \left( \frac{c_{\phi}^U}{1-\eta} \right)^{1-\eta} \left( \frac{h_{\phi}^U}{\eta} \right)^{\eta} \times N_{\phi} \right] + \sum_j c_j^O \right) dt,$$

where the first component is the aggregate utility of the employed workers, the second component is the aggregate utility of the unemployed workers, and the last component is the consumption of absentee landlords. Since the housing supply is fixed in each location, no additional social cost is incurred to the planner, no matter how the housing is allocated among the workers.

The planner chooses vacancy $V_\phi$ and size of unemployed worker $N_\phi$ for each $\phi$, along with housing and non-housing consumption for the workers and landlord $(c_\phi^E, c_\phi^U, h_\phi^E, h_\phi^U, c_j^O)$. The constraints the planner faces are:

1) Law of motion for employment for each $\phi$,

$$\dot{E}_\phi = m(N_\phi, V_\phi) - sE_\phi \tag{34}$$

where $\dot{E}_j^\chi$ is the evolution of employed worker

2) Land clearing for each location,

$$Q_j = \sum_\chi \left[ N_j h_j^{\chi,U} + E_j h_j^{\chi,E} \right] \tag{35}$$

3) Resource constraint of the planner,

$$0 = \sum_j Z_j + \sum_\phi (N_\phi b^\chi - k^\chi V_\phi) - \left[ \sum_\phi c_\phi^E \times E_\phi + c_\phi^U \times N_\phi \right] - \sum_j c_j^O \tag{36}$$

4) High-skill worker size and population constraints,

$$\xi = \sum_j E_j^s + N_j^s; \; 1 - \xi = \sum_j E_j^n + N_j^n \tag{37}$$

For each $\phi$, the size of the labor force $L_\phi$ equals the sum of the employed and the unemployed workers, i.e., $L_\phi = E_\phi + N_\phi$. The economy-wide resource constraint (equation 35) pins down the total level of consumption by absentee landlords. The derivation of the planner's solution is explained in more detail in Appendix B.2.

## 5.1 Comparison between Planner's and Decentralized Equilibrium

**Job Creation Condition**

Using the first order condition for $\theta_\phi$ and the equation for the co-state variable $u_\phi$, the planner's version of the job creation condition for each market $\phi$ is

$$\frac{k^\chi}{q(\theta_\phi)} = \frac{(1-\alpha)p_\phi y^\chi - [(1-\alpha)b^\chi + \alpha\theta_\phi k^\chi]}{r + s^\chi}, \tag{38}$$

whereas the decentralized job creation condition is

$$\frac{k^\chi}{q(\theta_\phi)} = \frac{(1-\beta)p_\phi y^\chi - [(1-\beta)b^\chi + \beta\theta_\phi k^\chi]}{r + s^\chi}. \tag{39}$$

Comparing the planner's job creation condition and the decentralized job creation condition within each market, one can easily see that the equivalence between them requires the following conditions,

$$\alpha = \beta,$$

where $\alpha$ is the matching function elasticity and $\beta$ is the bargaining power of workers. This is the within market Hosios [1990] condition, common in the random search literature. As in Şahin et al. [2014], imposing the standard Hosios [1990] condition eliminates within-market congestion externality for each market $\phi$.

### Spatial Optimality Condition

With multiple locations, the planner needs to choose how to allocate workers across locations. Using the first order condition for $N_\phi$, the planner's spatial optimality condition is

$$\overline{U}^{*\chi} = b^\chi + \frac{\alpha}{1-\alpha} k^\chi \theta_j^\chi \tag{40}$$

This condition states that the planner would allocate unemployed workers to a labor market until their contribution to locations is equalized. On the other hand, recall the decentralized spatial equilibrium condition equalizes the indirect utility of an unemployed worker across locations,

$$\overline{U}^\chi = \left(b^\chi + \frac{\beta}{1-\beta} k^\chi \theta_j^\chi\right) R_j^{-\eta}; \quad \forall j \in J, \quad \chi \in \{s, n\}. \tag{41}$$

The two expressions generally do not coincide. The addition of the housing market distorts the allocation since the planner and the unemployed worker have different valuations for residing in a location. The planner's unemployed worker allocation decision only concerns the effect an additional unemployed worker has on the market tightness, but the unemployed workers themselves care about not only the differences in tightness but also how the cost of living differs by location. The workers' indirect utility takes into account the housing cost, whereas the planner's optimal spatial condition does not. Therefore, even when the within-market standard Hosios [1990] condition ($\alpha = \beta$) is satisfied, the two spatial conditions coincide only when $\eta = 0$ or $R_j = R_{j'} = 0$.

Proposition 5 states the conditions when the decentralized equilibrium coincides with the planner's solution

**Proposition 5.** *The bargaining power parameters of workers $\beta_j^\chi$ need to satisfy the following conditions for the decentralized allocation to coincide with the constrained efficient allocation.*

1. *For the job creation conditions within each labor market to coincide* $\alpha_\phi = \beta_\phi$
2. *For the spatial equilibrium conditions to coincide*

$$\beta_j^\chi = 1 - \left[1 + \frac{R_j^\eta \left(b^\chi + \frac{\alpha}{1-\alpha} k^\chi \theta_j^\chi\right) - b^\chi}{k^\chi \theta_j^\chi}\right]^{-1}$$

*These conditions are simultaneously satisfied when $\eta = 0$ or $R_j = R_{j'}$ and $\alpha_\phi = \beta_\phi$.*

*Proof.* See appendix F.7 □

Note that the inefficiency is still caused by congestion externality. However, the addition of housing markets distorts the allocation even when within market Hosios [1990] condition is satisfied. Within market Hosios [1990] condition guarantees efficient job creation in the absence of housing market consideration since the housing supply is fixed, and housing is not directly related to the final good production. Nevertheless, the current utility function ties housing consumption to market tightness, which affects total output. Hence, even if the housing markets are frictionless, it complicates the existing congestion externality in the frictional labor market via the inseparability of job finding and housing consumption location, making the competitive housing market relevant.

In random search models, inefficiency arises due to the missing price of market tightness. One way to implement the Hosios [1990] condition and to restore efficiency is charging an entry fee to workers such that the cost of participating in the labor market is equal to the cost of the congestion they create for other workers. In my model, however, the "entry fee" workers pay to participate is the housing rent due to the inseparability of work and home location. Yet, the size of the housing rent does not equal the price of market tightness. Therefore, the market tightness is mispriced, leading to discrepancies between the planner's equilibrium conditions and the decentralized equilibrium conditions.

Additionally, the two skill levels further complicate the problem since two market tightnesses collectively affect the common housing rent, and despite differences in contribution to output, high-skill, and low-skill workers face the same price to enter the location. The expected cost and benefit of being in a local labor market are further distorted between the decentralized equilibrium and the planner's solution. Therefore, the common housing market forces two market tightnesses to affect each other, even if the within market standard Hosios [1990] condition is satisfied within each skill-location labor market $\phi$, the between-skill interactions of the market tightness still leads to misallocation since the within market Hosios [1990] condition only eliminate within-market congestion by equating the costs of congestion and benefits of participation within a local labor market; therefore, even when satisfied, the additional congestion cost from the housing market distorts the de facto cost of congestion. The cost of congestion no longer equals the benefit the high-skill worker's participation generates, but it equals the cost of congestion their participation generates plus the change in housing prices due to their participation. Therefore, the market tightness is still mispriced by the housing rent.

Workers of both skill types have incentives to locate in the more productive location; they will do so in the absence of the housing market. However, the common local housing market disproportionally discourages low-skill workers from living in more productive locations. It allocates more workers of both skill types to the less-productive location, leading to inefficiency.

As shown in Proposition 5, with the current common housing market, only when $\eta = 0$ or $R_j = R_{j'}$ could the within market Hosios [1990] condition restore efficiency.

# 6 Quantitative Analysis

This section presents the calibrated version of the model to compare the decentralized and constrained efficient allocations, compare the frictional labor market with the competitive labor market, and perform counterfactual policy experiments. First, I introduce the data used for the quantitative exercises in Section 6.1. In Section 6.2, I introduce a few modifications to the model that are unique to its quantitative version. Section 6.3 details the calibration strategy. Section ?? studies the effects search frictions have on the Great Divergence. Section 6.5 compares the decentralized and constrained efficient allocations. Lastly, Section 6.6 performs policy experiments.

### 6.1 Data

The model is calibrated to a representative high-skill-intensive location $H$, using data from San Francisco-Oakland-Hayward MSA, and a representative low-skill-intensive location $L$, using data from Detroit-Warren-Dearborn MSA. The period is from 2005 to 2019. The primary data set used for the quantitative exercises is the American Community Survey (ACS), obtained from IPUMS Steven Ruggles and Sobek [Accessed Aug 1st, 2022].

In the quantitative version of the model, high-skill versus low-skill workers are defined based on the worker's occupation, using the task index created by Autor and Dorn (2013). For each occupation, I construct a skill index $AM$ for each occupation $k$, which is defined as the following:

$$AM_k = \frac{\left(T^A_{k,1980} - T^M_{k,1980}\right) - \underline{AM}}{\overline{AM} - \underline{AM}},$$

where $T^A_{k,1980}$ is abstract task input, defined as the average of the Dictionary of Occupational Titles (DOT) variable for "direction control and planning" which measures managerial and interactive tasks and "GED Math", measuring mathematical and formal reasoning requirements. $T^M_{k,1980}$ is the manual task input, defined as the DOT variable for an occupation's demand for "eye-hand-foot coordination". The AM index's goal is to capture each occupation's skill level. $\underline{AM}$ and $\overline{AM}$ are defined as follows for normalization purposes

$$\underline{AM} \equiv \min\left\{T^A_{1,1980} - T^M_{1,1980}, \ldots, T^A_{K,1980} - T^M_{K,1980}\right\}$$

$$\overline{AM} \equiv \max\left\{T^A_{1,1980} - T^M_{1,1980}, \ldots, T^A_{K,1980} - T^M_{K,1980}\right\}$$

If $AM_k > 0.618$, occupation $k$ is considered a high-skill occupation; otherwise, $k$ is considered a low-skill occupation. Using this categorization, I find the share of high-skill workers in the sample is $\xi = 0.4513$. More information about AM can be found in appendix D.

### 6.2 Quantitative version of the model

I introduce two differences in the quantitative version of the model compared to the environment in Section 2. First, I generalize the model by endogenizing the job destruction decision. Worker productivity becomes idiosyncratic and is drawn from a distribution $F_\phi$ that depends on the location and skill pair. Firms optimally choose reservation productivity $y^*_\phi$ and destroy jobs with productivity less than it, where the value of a filled job with reservation productivity equals zero. At rate $\lambda$, employed workers re-draw their productivity. If the newly drawn productivity is less than the reservation productivity, the match is destroyed, the worker becomes unemployed, and the firm becomes vacant.

This extension preserves the basic structure of the equilibrium presented in Section 3. The main differences are the following. First, the equilibrium has an additional element, reservation productivity

$y^*_\phi$. Second, an additional equilibrium condition, the Job Destruction condition, is introduced. The Job Creation condition and the Job Destruction condition jointly pin down the reservation productivity and the market tightness for each skill location pair. Third, the variation in unemployment comes from differences in the job finding rate and the endogenous separation rate. Since the critical condition for worker allocation, the spatial equilibrium condition, does not explicitly involve reservation productivity in the extended model, it is identical to the spatial equilibrium condition presented in Section 3. The analytical results from Section 3 hold.

Additionally, I allow matching efficiency parameter $A_j$ to differ by location and the unemployment benefit $b_\phi$ to be different for each skill-location group. Flexibility in these parameters allows the calibration to be more precise. Details and derivation of the quantitative version of the model can be found in appendix C.

## 6.3 Calibration

The calibration uses the following parameters from the literature. The rate of productivity shock is set to be $\lambda = 0.085$, following Fujita and Ramey [2012]. Following Petrongolo and Pissarides [2001], the elasticity of the matching function is set to $\alpha = 0.5$, which is in line with empirical evidence. The worker's bargaining power is then set to $\beta = 0.5$ to implement the Hosios [1990] condition. Following Krusell et al. [2000], the elasticity of substitution between high-skill and low-skill workers is set to $\rho = 0.4$.

From the data sample, the total share of high-skill workers in the two locations is $\xi = 0.4513$. The discount rate is r= 0.0143, the average annual interest rate during the period. I can find the average market tightness for each location using the US Job Openings and Labor Turnover Survey (JOLTS) MSA level data from January 2005 to December 2019. Since the expression of job finding rate is $f(\theta_\phi) = A_j\theta_\phi^\alpha$, $A_j$ can be backed out where $A_H = 0.74$ and $A_L = 0.67$. Following the affordable housing guideline (Health and Code [1977]), I use 30 percent as the share of income spent on housing, $\eta = 0.3$.

I use the land area as a proxy for the housing supply in each location. I normalized the land area of location $H$ to be 1. Census Bureau's data of land areas indicates that the land area in $L$ is 57% bigger than the land area in H; therefore, $Q_L = 1.57$. Following Krusell et al. [2000] and using the high-skill labor income share for each location, the weight of high-skill workers in the final goods production function is $\sigma_H = 0.648$ and $\sigma_L = 0.476$. Productivities of both skill types are assumed to follow Pareto Distributions $F \sim Pareto(y_{m,\phi}, \alpha_\phi)$ where $y_{m,\phi}$ is the scale parameter for the skill location group and $\alpha_\phi$ is its shape parameter. Since the scale parameter in the Pareto distribution reflects the lower bound of the distribution, it is obtained from the minimum level of schooling of each skill location group, where $y^s_{m,H} = 1.2,\ y^n_{m,H} = 0.5,\ y^s_{m,L} = 1.1,\ y^n_{m,L} = 0.5$. [6] The shape parameter is calibrated by using the mean wage generated by the model to back out the mean productivity for each skill location group, where the expression of the mean productivity involves only the shape and scale parameter of the Pareto distribution. The results are $\alpha^s_H = 1.28, \alpha^n_H = 1.45, \alpha^s_L = 1.2$ and $\alpha^n_L = 1.43$. Table 4 summarizes the parameter values.

[6] The minimum schooling level for high-skill workers in location H, low-skill workers in location H, high-skill workers in location L and low-skill worker in location L are twelve years, five years, eleven years and five years respectively.

Table 4: Parameter Value

| Parameter | | Value | Source |
|---|---|---|---|
| I. From Literature | | | |
| Matching function elasticity | $\alpha$ | 0.5 | Petrongolo and Pissarides [2001] |
| Worker bargaining power | $\beta$ | 0.5 | Hosios [1990] Efficiency Condition |
| Productivity shock | $\lambda$ | 0.085 | Fujita and Ramey [2012] |
| Elasticity of substitution | $\frac{1}{1-\rho}(\rho)$ | 1.67(0.4) | Krusell et al. [2000] |
| Share of spending on housing | $\eta$ | 0.3 | Health and Code [1977] |
| II. From Data | | | |
| Discounting rate | r | 0.0143 | Annual federal funds rate |
| Total share of skilled labor | $\xi$ | 0.4513 | Share of high-skill occupation, ACS |
| High-skill worker weight in $H$ | $\sigma_H$ | 0.648 | High-skill labor income share |
| High-skill worker weight in $L$ | $\sigma_L$ | 0.476 | High-skill labor income share |
| Matching Efficiency in location H | $A_H$ | 0.74 | Job Finding Prob. in H, JOLTS |
| Matching Efficiency in location L | $A_L$ | 0.67 | Job Finding Prob. in L, JOLTS |
| Pareto dist. scale parameter (H, high-skill) | $y^s_{H,m}$ | 1.2 | Minimum schooling level, ACS |
| Pareto dist. scale parameter (H, low-skill) | $y^n_{H,m}$ | 0.5 | Minimum schooling level, ACS |
| Pareto dist. scale parameter (L, high-skill) | $y^s_{L,m}$ | 1.1 | Minimum schooling level, ACS |
| Pareto dist. scale parameter (L, low-skill) | $y^n_{L,m}$ | 0.5 | Minimum schooling level, ACS |
| Land area in location $H$ | $T_H$ | 1 | Normalization |
| Land area in location $L$ | $T_L$ | 1.57 | Census Bureau |

Table 5: Calibrated Parameters

| Parameter | | Calibrated Value |
|---|---|---|
| Unemployment utility (high-skill, location H) | $b_H^s$ | 0.832 |
| Unemployment utility (low-skill, location H) | $b_H^n$ | 0.383 |
| Unemployment utility (high-skill, location L) | $b_L^s$ | 0.799 |
| Unemployment utility (low-skill, location L) | $b_L^n$ | 0.369 |
| Flow vacancy cost (high-skill) | $k^s$ | 1.95 |
| Flow vacancy cost (low-skill) | $k^n$ | 0.98 |
| Shape parameter of Pareto dist. (high-skill, location H) | $a_H^s$ | 1.38 |
| Shape parameter of Pareto dist. (low-skill, location H) | $a_H^n$ | 1.5 |
| Shape parameter of Pareto dist. (high-skill, location L) | $a_L^s$ | 1.3 |
| Shape parameter of Pareto dist. (low-skill, location L) | $a_L^n$ | 1.5 |
| Upper bound for productivity (high-skill, location H) | $\bar{y}_H^s$ | 91.49 |
| Upper bound for productivity (low-skill, location H) | $\bar{y}_H^n$ | 54.68 |
| Upper bound for productivity (high-skill, location L) | $\bar{y}_L^s$ | 67.10 |
| Upper bound for productivity (low-skill, location L) | $\bar{y}_L^n$ | 46.76 |

The remaining parameters are calibrated as follows. Unemployment insurance $b_\phi$ for each skill location group is calibrated using the replacement rate, where $\frac{b_\phi}{\bar{w}_\phi} = 0.71$, following Hall and Milgrom [2008], where $\bar{w}_\phi$ is the average wage for each skill location group generated by the model. Flow vacancy cost is calibrated to match its share of average labor productivity for each skill level, following Hagedorn and Manovskii [2008]. I use the mean-min (Mm) wage ratio for each skill location group to calibrate the shape parameter of the Pareto distribution where $a_H^s = 1.38$, $a_H^n = 1.5$, $a_L^s = 1.3$, $a_L^n = 1.5$. Lastly, I used the $90-10$ percentile wage ratio for each skill location group to calibrate the upper bound of match-specific productivity by skill location group, where $\bar{y}_H^s = 91.49, \bar{y}_H^n = 54.68, \bar{y}_L^s = 67.1$ and $\bar{y}_L^n = 46.76$. The results of the calibrated parameters are summarized in Table 5. Table 6 illustrates that the model closely matches the empirical targets.

Table 6: Targeted Moments

| | Data | Model | | Data | Model |
|---|---|---|---|---|---|
| Replacement Rate (Hs) | 0.71 | 0.710 | Replacement Rate (Ls) | 0.71 | 0.710 |
| Replacement Rate (Hn) | 0.71 | 0.709 | Replacement Rate (Ln) | 0.71 | 0.709 |
| Mm wage ratio(Hs) | 2.498 | 2.467 | Mm wage ratio(Ls) | 2.479 | 2.461 |
| Mm wage ratio(Hn) | 2.502 | 2.491 | Mm wage ratio(Ln) | 2.532 | 2.524 |
| 90-10 percentile ratio (Hs) | 7.946 | 7.951 | 90-10 percentile ratio (Ls) | 6.903 | 6.906 |
| 90-10 percentile ratio (Hn) | 12.79 | 12.79 | 90-10 percentile ratio (Ln) | 12.64 | 12.64 |

Table 7: Non-Targeted Moments

| | | Data | Model |
|---|---|---|---|
| (a) Labor Market Composition | | | |
| Population share of high-skill location | $L_H$ | 0.6043 | 0.5384 |
| Share of high-skill worker in high-skill place | $\zeta_H$ | 0.518 | 0.5828 |
| Share of high-skill worker in low-skill place | $\zeta_L$ | 0.3481 | 0.2979 |
| (b) Unemployment Ratio | | | |
| Unemployment rate Δ% for high-skill worker | $(u^s_H - u^s_L)/u^s_L$ | -14.35% | -21.7% |
| Unemployment rate Δ% for low-skill worker | $(u^n_H - u^n_L)/u^n_L$ | -30.12% | -21.5% |

To assess the model's performance, I look at several non-targeted empirical moments that are believed to be particularly important for the model. First, I look at the composition of labor markets. Panel (a) of Table 7 compares labor market compositions between the data and the model. The model predicts 53.84% of the workers are in location H; among them, 58.28% are high-skill workers. 29.79% of the labor force in location L are high-skill workers. The model performs well in matching the labor market compositions in the data as we can see that the difference between the data and the model are narrow for $L_H, \zeta_H$, and $\zeta_L$. Panel (b) of Table 7 compares the unemployment ratio between locations. For both skill groups, the model predicts that the unemployment rate is lower in location $H$ than in location $L$. Overall, the calibration matches the labor force composition and the relationship between locations for wages and unemployment rates, as we have seen in descriptive facts presented in Section 1.1.

### 6.4 Search Frictions and the Great Divergence

This section quantitatively assesses the effect of labor market search friction on the great divergence. As discussed in Section 4, in the model with competitive labor markets, the spatial equilibrium conditions, equation **??** indicate that workers' location choices are based on the relative sizes of wages and housing prices. However, in the case of labor markets with search frictions, what determines the spatial equilibrium are the marginal product of labor, market tightness, and housing prices, as shown in **??**. The presence of market tightness in the spatial equilibrium condition indicates that the allocation of workers will be different for the frictional labor market and the competitive labor market. Table 8 summarizes the allocation for the two different labor markets.

Table 8: Comparison of allocations

| | Competitive Labor Market | Frictional Labor Market | %Diff |
|---|---|---|---|
| Share of high-skill worker in H | 0.5917 | 0.5839 | 1.3349% |
| Labor force in H | 0.5070 | 0.5376 | -5.6911% |
| Share of high-skill worker in L | 0.3069 | 0.2971 | 3.2933% |
| Labor force in L | 0.4930 | 0.4624 | 6.6167% |
| Location wage ratio (high-skill workers) | 1.3423 | 1.0542 | 27.3292% |
| Location wage ratio (low-skill worker) | 1.3423 | 1.0417 | 28.8634% |
| Location housing price ratio | 2.6681 | 2.3487 | 13.5994% |

Compared to the frictional model, the competitive labor market equilibrium places a higher share of high-skill workers but fewer workers in location $H$. The location wage gap is also higher in the competitive labor market model. It is 27.3% higher for high-skill workers and 28.9% higher for low-skill workers. Again, in the competitive model, wages for both skill types in H must be much higher to attract workers there. Lastly, the location rent gap in the competitive model is 13.6% higher than in the frictional model, resulting from the bigger location wage gaps. Therefore, we can say that the model with labor market search friction moderates the great divergence. Lastly, a back-of-the-envelope calculation for the utility of high- and low-skill workers suggests that the utility gap for high-skill workers and low-skill workers in the competitive labor market is about 4% bigger than in the model with frictional labor markets.

## 6.5 Planner's vs. decentralized allocation

In this section, I compare the constrained efficient and decentralized allocations under the parameters and calibrated parameters presented in Table 4 and Table 5. For the constrained efficient allocation, I compute the planner's choice of reservation productivity $y^*_\phi$, market tightness $\theta_\phi$, number of workers $L_\phi$ to maximize the sum of steady-state net output of location $H$ and $L$. Table 9 summarizes the results.

Column (1) of Table 9 shows the decentralized allocation. For both high-skill and low-skill workers, the unemployment rate is higher in location $L$. Within each location, the unemployment rate of high-skill workers is lower than the unemployment rate of low-skill workers. The pattern of unemployment rates matches descriptive facts presented in Figure 2. Regarding the distribution of workers, the calibrated model suggests that location $H$ has more high-skill workers than low-skill workers. In contrast, location $L$ has more low-skill workers than high-skill workers.

Column (3) of Table 9 shows the percentage differences between decentralized and constrained efficient allocations. The constrained efficient allocation exhibits a higher level of reservation productivity for both groups in both locations. The planner allocates more workers of both skill types to location H. For high-skill workers in both locations, the constrained efficient allocation shows higher market tightness relative to the decentralized version, whereas the reverse is true for low-skill workers. Finally, the aggregate output of the constrained efficient allocation is 4.794% higher than the decentralized allocation.

Table 9: Allocation Comparison

| | | (1) Decentralized | (2) Centralized | (3) % Difference |
|---|---|---|---|---|
| **Reservation Productivity** | | | | |
| | $y_H^{s*}$ | 4.7017 | 5.9492 | 26.5326% |
| | $y_H^{n*}$ | 1.6961 | 2.0840 | 22.8709% |
| | $y_L^{s*}$ | 3.9492 | 5.3072 | 34.3857% |
| | $y_L^{n*}$ | 1.4454 | 1.5488 | 7.1504% |
| **Market Tightness** | | | | |
| | $\theta_H^s$ | 2.3340 | 3.2395 | 38.7986% |
| | $\theta_H^n$ | 2.0679 | 1.9374 | -6.3074% |
| | $\theta_L^s$ | 1.7284 | 3.2112 | 85.7887% |
| | $\theta_L^n$ | 1.5276 | 1.5768 | 3.2247% |
| **Distribution of workers** | | | | |
| | $L_H$ | 0.5376 | 0.8659 | 61.0711% |
| | $\zeta_H$ | 0.5839 | 0.4986 | -14.6111% |
| | $L_L$ | 0.4624 | 0.1341 | -71.0027% |
| | $\zeta_L$ | 0.2971 | 0.1458 | -50.9410% |
| **Unemployment Rate** | | | | |
| | $u_H^s$ | 0.0599 | 0.0538 | -10.3179% |
| | $u_H^n$ | 0.0629 | 0.0679 | 7.9636% |
| | $u_L^s$ | 0.0725 | 0.0581 | -19.9231% |
| | $u_L^n$ | 0.0756 | 0.0762 | 0.8310% |
| **Output** | | | | |
| | $Z_H + Z_L$ | 0.3775 | 0.3956 | 4.794% |

The discrepancies between the decentralized and constrained efficient outcomes arise from the inefficiencies discussed in Section 5. [7] Without considering housing costs, more low-skill workers moved into location $H$. Hence $L_H$ increased, and $\zeta_H$ decreased. Leading to a higher low-skill unemployment rate in location $H$. Therefore, compared to the constrained efficient equilibrium, the decentralized equilibrium allocates inefficiently small amounts of workers of both skill types in location $H$.

## 6.6 Policy Experiments

In this section, I study the effects of policies that aim at correcting the inefficiencies caused by the externalities. Table 10 contains the results of the experiment, where Column (1) is the allocation of the decentralized equilibrium, Column (2) shows results from the counterfactual experiment, and Column (3) compare the difference between the decentralized equilibrium and the allocation with the policy.

Table 10: Policy Experiments

| | | (1) Benchmark | (2) Worker<br>Level | subsidy<br>% Difference |
|---|---|---|---|---|
| **Reservation Productivity** | | | | |
| | $y_H^{s*}$ | 4.7017 | 4.6959 | -0.1242% |
| | $y_H^{n*}$ | 1.6961 | 1.6902 | -0.3486% |
| | $y_L^{s*}$ | 3.9492 | 3.9419 | -0.1859% |
| | $y_L^{n*}$ | 1.4454 | 1.4413 | -0.2848% |
| **Unemployment rates** | | | | |
| | $u_H^s$ | 0.0599 | 0.0599 | -0.0291% |
| | $u_H^n$ | 0.0629 | 0.0629 | 0.0809% |
| | $u_L^s$ | 0.0725 | 0.0725 | 0.0442% |
| | $u_L^n$ | 0.0756 | 0.0755 | -0.1175% |
| **Worker Distribution** | | | | |
| | $L_H$ | 0.5376 | 0.5394 | 0.3366% |
| | $\zeta_H$ | 0.5839 | 0.5826 | -0.2321% |
| | $L_L$ | 0.4624 | 0.4606 | -0.3942% |
| | $\zeta_L$ | 0.2971 | 0.2976 | 0.1523% |
| **Aggregate Welfare** | | 0.3563 | 0.3565 | 0.05613% |

[7] Note that even though the baseline model which Section 5 is based on is different from the endogenous separation version of the model that the quantitative exercises are based on, the intuition of inefficiencies are similar and is a result of the differences in the spatial equilibrium condition.

### 6.6.1 Low-skill worker relocation subsidy

Since Table 9 shows that inefficiently low numbers of workers, in particular, low-skill workers, choose location $H$, the first policy experiment studies the effect of lump-sum subsidy for low-skill workers in location H. A fixed subsidy $\tau^m$ is given to all low-skill workers in location H regardless of employment status. The subsidies are financed by a lump-sum tax $\tau^c$ on all workers, regardless of employment status, skill type, or location. The subsidy's size equals 10 percent of housing spending an unemployed low-skill worker in location H would pay [8].

As seen in column (2), when low-skills workers in location H are subsidized by all firms, the labor force increases in location $H$, and it becomes slightly less concentrated in high-skill workers. The allocation of worker distribution is moving toward the constrained efficient allocation. Compared to the benchmark decentralized allocations, this policy experiment creates more jobs in location $H$, and unemployment rates are lower for all skill-location groups. Putting equal weights on all skill-location groups, the aggregate welfare is 0.05613% higher under this policy experiment.

## 7 Conclusion

This paper documents the geographic dispersion of unemployment rates in the US for workers of different skill levels. I then develop a model featuring frictional labor markets in a spatial equilibrium to study how the frictional labor market shapes the great divergence across US cities and its effect on the optimal allocation of heterogeneous workers. The model generates theoretical results that explain the empirical pattern of wages and unemployment rates for high- and low-skill workers and the skill composition across labor markets.

Comparing the model with labor market frictions with the model with competitive labor markets shows that frictional labor markets moderate the divergence in high-skill worker concentration and the wage gap between locations compared to its full employment counterpart. The high-wage location also features low unemployment rates, particularly for low-skill workers. A bigger wage gap is required to obtain the spatial equilibrium without friction in the labor market. A normative analysis shows that the decentralized equilibrium is never efficient even if the standard within market Hosios [1990] condition holds, but can be efficient if a generalized version of the Hosios [1990] condition holds. The additional inefficiency is caused by distortions resulting from the housing market since the housing rent takes on the additional role of an entry fee to labor markets but is not priced accordingly. A calibrated version of the model using representative high-skill-intensive and low-skill-intensive locations shows that inefficient amounts of workers of both skill types choose to stay in the low-skill-intensive location due to the high housing cost of the high-skill-intensive location. Additionally, the amount of jobs created in high-skill-intensive locations is inefficiently low for both skill types. Subsidies incentivizing workers to locate to high-skill-intensive locations raise aggregate welfare.

[8] Details of the policy experiment equilibrium can be found in appendix E.

# Appendices

## A Descriptive facts with alternative definition of skill

Figure 6: Changes in Share of High-Skill Workers and Unemployment Rates by Skill Types, 2005-2019

(a) (b)

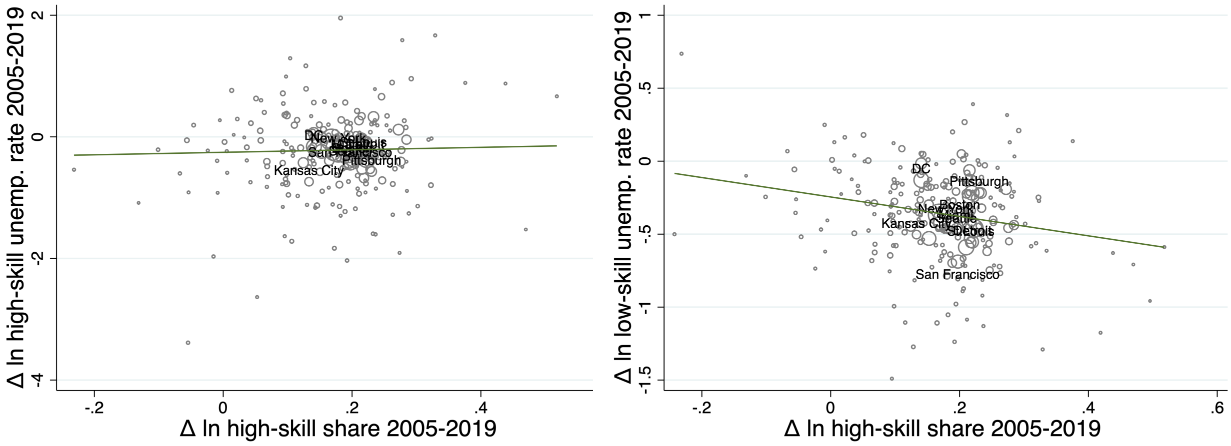



Figure 7: Changes in Unemployment and Nominal Wages by Skill Types, 2005-2019

(a) (b)

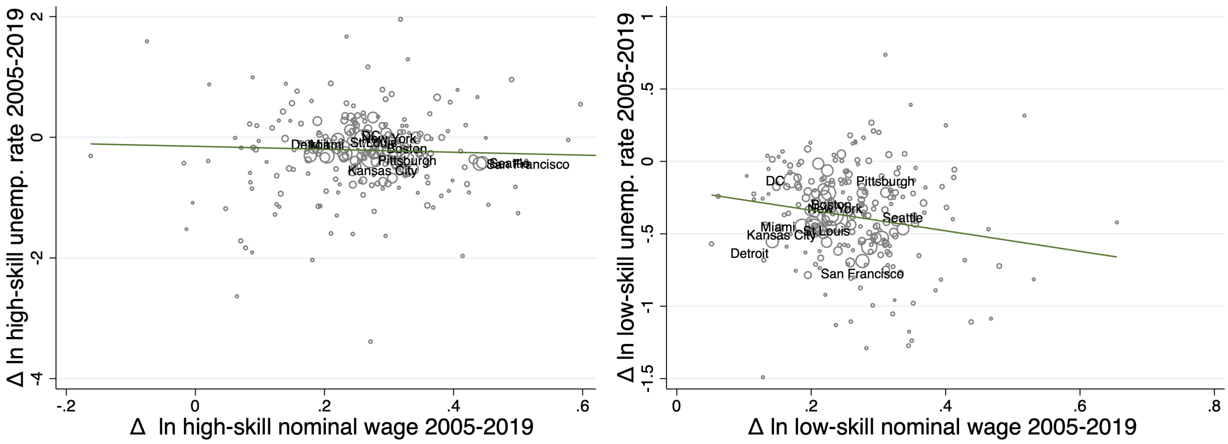
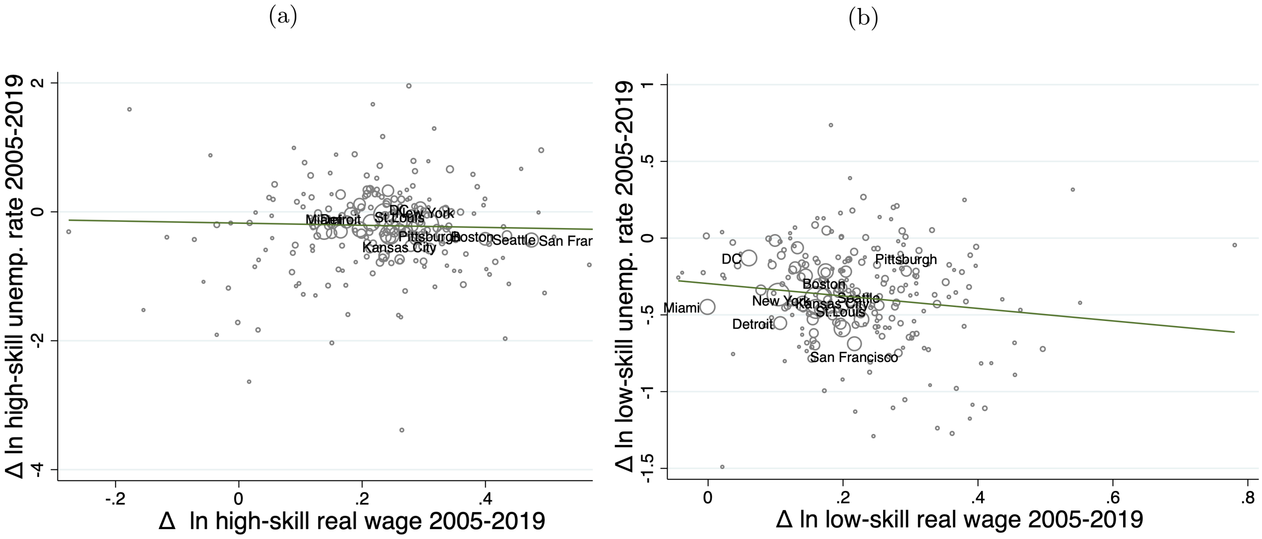


Figure 8: Changes in Unemployment and Real Wages by Skill Types, 2005-2019



## A.1 Tables

Table 11: Share of High-Skill Worker and Unemployment Rates

| Log Unemployment Rate | (1) High-Skill OLS | (2) Low-Skill IV | (3) High-Skill OLS | (4) Low-Skill IV |
|---|---|---|---|---|
| Log Share of High-Skill Worker (Educ) | 0.0330 | 0.124* | -0.152*** | -0.0200 |
| | (0.0377) | (0.0646) | (0.0210) | (0.0360) |
| Observations | 2,622 | 2,563 | 2,643 | 2,583 |
| R-squared | 0.295 | 0.295 | 0.459 | 0.458 |
| MSA FE | Yes | Yes | Yes | Yes |
| Year FE | Yes | Yes | Yes | Yes |

Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

# B Derivation

## B.1 Equilibrium Derivation from section 2

### Consumption and housing decision

Worker's maximization problem is

$$\max_{c_\phi, h_\phi} \mathcal{U} = \left(\frac{c_\phi}{1-\eta}\right)^{1-\eta}\left(\frac{h_\phi}{\eta}\right)^{\eta},$$

$$s.t. \;\; w_\phi = c_\phi + R_j h_\phi.$$

First order conditions wrt $(h_\phi, c_\phi)$ are

$$\frac{\partial H}{\partial c^E_\phi} = 0, \; \frac{\partial H}{\partial c^U_\phi} = 0 \Rightarrow E_\phi(\frac{c^E_\phi}{h^E_\phi}\frac{\eta}{1-\eta})^{-\eta} - E_\phi = 0; \;\; N_\phi(\frac{c^U_\phi}{h^U_\phi}\frac{\eta}{1-\eta})^{-\eta} - N_\phi = 0; \tag{42}$$

$$\frac{\partial H}{\partial h^U_\phi} = 0, \frac{\partial H}{\partial h^U_\phi} = 0 \Rightarrow E_\phi(\frac{c^E_\phi}{h^E_\phi}\frac{\eta}{1-\eta})^{1-\eta} - \kappa_j E_\phi = 0; \;\; N_\phi(\frac{c^U_\phi}{h^U_\phi}\frac{\eta}{1-\eta})^{1-\eta} - \kappa_j N_\phi = 0; \tag{43}$$

$$\frac{\partial H}{\partial c^E_\phi} = 0 \Rightarrow \frac{1-\eta}{c^E_\phi}\mathcal{U}^E_\phi E_\phi - E_\phi = 0 \qquad \frac{\partial H}{\partial c^U_\phi} = 0 \Rightarrow \frac{1-\eta}{c^U_\phi}\mathcal{U}^U_\phi N_\phi - N_\phi = 0; \tag{44}$$

$$\frac{\partial H}{\partial h^U_\phi} = 0 \Rightarrow \frac{\eta}{h^E_\phi}\mathcal{U}^E_\phi E_\phi - \kappa_j E_\phi = 0; \qquad \frac{\partial H}{\partial h^U_\phi} = 0 \Rightarrow \frac{\eta}{h^U_\phi}\mathcal{U}^U_\phi N_\phi - \kappa_j N_\phi = 0. \tag{45}$$

Hence $\kappa_j = (\frac{c^E_\phi}{h^E_\phi}\frac{\eta}{1-\eta}) = (\frac{c^U_\phi}{h^U_\phi}\frac{\eta}{1-\eta})$ The first two F.O.C. leads to the following equation

$$h^E_\phi = \frac{c^E_\phi}{\kappa_j}\frac{\eta}{1-\eta}; \;\; h^U_\phi = \frac{c^U_\phi}{\kappa_j}\frac{\eta}{1-\eta}; \;\; \mathcal{U}^E_\phi = \frac{c^E_\phi}{1-\eta}; \;\; \mathcal{U}^U_\phi = \frac{c^U_\phi}{1-\eta}.$$

## B.2 Planner's problem from section 5

The social planner aims to maximize a social welfare function subject to resource constraints and the law of motion of employment. The social welfare function puts equal welfare weights for the three groups of agents: two types of workers and absentee landlords. Let $N_\phi$ denote the number of unemployed workers of type $\phi$, and let $E_\phi$ denote the number of employed workers of type $\phi$.

The planner's objective function is

$$\omega = \int_0^\infty e^{-rt}\left(\sum_\phi \left[(\frac{c^E_\phi}{1-\eta})^{1-\eta}(\frac{h^E_\phi}{\eta})^\eta \times E_\phi + (\frac{c^U_\phi}{1-\eta})^{1-\eta}(\frac{h^U_\phi}{\eta})^\eta \times N_\phi\right] + \sum_j c^O_j\right) dt,$$

The first component is the aggregate utility of the employed workers, the second component is the aggregate utility of the unemployed workers, and the last component is the consumption of absentee landlords.

Planner chooses vacancy number $V_\phi$ and number of unemployed workers $N_\phi$, for each $\phi$, along with housing and non-housing consumption for the workers and landlord $(c^E_\phi, c^U_\phi, h^E_\phi, h^U_\phi, c^O_j)$. The constraints the planner faces are [1] the law of motion for employment for each $\phi$, [2] land clearing for each location, [3] the resource constraint of the planner, and [4] high-skill worker size and population constraints,

$$\text{LOM of employed worker} \quad \dot{E}_j^{\chi} = m(N_\phi, V_\phi) - sE_\phi$$

$$\text{Local housing constraint} \quad Q_j = \left[\sum_{\chi} N_j h_j^{\chi,U} + E_j h_j^{\chi,E}\right]$$

$$\text{Resource Contraint} \quad \sum_j Z_j + \sum_\phi (N_\phi b^\chi - k^\chi V_\phi) - \left[\sum_\phi c_\phi^E \times E_\phi + c_\phi^U \times N_\phi\right] - \sum_j c_j^O = 0$$

$$\text{Total workers constraint} \quad \xi = \sum_j E_j^s + N_j^s;\ 1 - \xi = \sum_j E_j^n + N_j^n$$

The current-value Hamiltonian for the planner is

$$\mathcal{H}(E_\phi, N_\phi, V_\phi, c_\phi^E, c_\phi^U, h_\phi^E, h_\phi^U, \gamma_\phi, \mu_j, \phi^\chi) = \sum_\phi \left[(\frac{c_\phi^E}{1-\eta})^{1-\eta}(\frac{h_\phi^E}{\eta})^\eta \times E_\phi + (\frac{c_\phi^U}{1-\eta})^{1-\eta}(\frac{h_\phi^U}{\eta})^\eta \times N_\phi - \left(c_\phi^E \times E_\phi + c_\phi^U \times N_\phi\right)\right]$$

$$+ \sum_j Z_j + \sum_\phi (N_\phi b^\chi - k^\chi V_\phi) + \sum_\phi \gamma_\phi \left[m(N_\phi, V_\phi) - sE_\phi\right] + \mu_j \left[Q_j - \left(\sum_\chi N_j h_j^{\chi,U} + E_j h_j^{\chi,E}\right)\right]$$

$$+ \psi^s \left[\xi - \left(\sum_j E_j^s + N_j^s\right)\right] + \psi^n \left[1 - \xi - \left(\sum_j E_j^s + N_j^s\right)\right]$$

where $E_\phi$ are the state variables, $(N_\phi, V_\phi, c_\phi^E, c_\phi^U, h_\phi^E, h_\phi^U)$ are control variables, and $(\gamma_\phi, \mu_j, \phi^\chi)$ are the co-state variables.

**Optimal consumption and housing**

First order conditions wrt $(h_\phi, c_\phi)$ are

$$\frac{\partial H}{\partial c_\phi^E} = 0,\ \frac{\partial H}{\partial c_\phi^U} = 0 \Rightarrow E_\phi(\frac{c_\phi^E}{h_\phi^E}\frac{\eta}{1-\eta})^{-\eta} - E_\phi = 0;\quad N_\phi(\frac{c_\phi^U}{h_\phi^U}\frac{\eta}{1-\eta})^{-\eta} - N_\phi = 0 \tag{46}$$

$$\frac{\partial H}{\partial h_\phi^U} = 0,\ \frac{\partial H}{\partial h_\phi^U} = 0 \Rightarrow E_\phi(\frac{c_\phi^E}{h_\phi^E}\frac{\eta}{1-\eta})^{1-\eta} - \mu_j E_\phi = 0;\quad N_\phi(\frac{c_\phi^U}{h_\phi^U}\frac{\eta}{1-\eta})^{1-\eta} - \mu_j N_\phi = 0 \tag{47}$$

$$\frac{\partial H}{\partial c_\phi^E} = 0 \Rightarrow \frac{1-\eta}{c_\phi^E}\mathcal{U}_\phi^E E_\phi - E_\phi = 0 \qquad \frac{\partial H}{\partial c_\phi^U} = 0 \Rightarrow \frac{1-\eta}{c_\phi^U}\mathcal{U}_\phi^U N_\phi - N_\phi = 0 \tag{48}$$

$$\frac{\partial H}{\partial h_\phi^U} = 0 \Rightarrow \frac{\eta}{h_\phi^E}\mathcal{U}_\phi^E E_\phi - \mu_j E_\phi = 0 \qquad \frac{\partial H}{\partial h_\phi^U} = 0 \Rightarrow \frac{\eta}{h_\phi^U}\mathcal{U}_\phi^U N_\phi - \mu_j N_\phi = 0 \tag{49}$$

Hence $\mu_j = (\frac{c_\phi^E}{h_\phi^E}\frac{\eta}{1-\eta}) = (\frac{c_\phi^U}{h_\phi^U}\frac{\eta}{1-\eta})$. The first two F.O.C.s lead to the following equation

$$h_\phi^E = c_\phi^E\frac{\eta}{1-\eta};\quad h_\phi^U = c_\phi^U\frac{\eta}{1-\eta};\quad \mathcal{U}_\phi^E = \frac{c_\phi^E}{1-\eta};\quad \mathcal{U}_\phi^U = \frac{c_\phi^U}{1-\eta}$$

therefore $\left(\mathcal{U}_\phi^E - \mu_j h_\phi^E - c_\phi^E\right) = 0$, $\left(\mathcal{U}_\phi^U - \mu_j h_\phi^U - c_\phi^U\right) = 0$

**FOC wrt $V_\phi$**

$$0 = -k^\chi + \gamma_\phi \frac{\partial m(N_\phi, V_\phi)}{\partial V_\phi}$$

Therefore, $\gamma_\phi = \frac{k^\chi}{(1-\alpha)AN_\phi^\alpha V_\phi^{-\alpha}} = \frac{k^\chi}{(1-\alpha)q(\theta_\phi)}$, where $\psi^\chi$ is the shadow value of an additional worker of skill level $\chi$ in the unemployment pool regardless of location.

**Co-state equation for $E_\phi$**

$$\frac{\partial H}{\partial E_\phi} = r\gamma_\phi - \dot{\gamma}_\phi \Rightarrow r\gamma_\phi - \dot{\gamma}_\phi = -\gamma_\phi s + \frac{\partial Z_j}{\partial E_\phi} - \psi^\chi + \left(\mathcal{U}_\phi^E - \mu_j h_\phi^E - c_\phi^E\right)$$

impose steady state condition $\dot{\gamma_\phi} = 0$, and plug in optimal housing consumption the expression becomes

$$\gamma_\phi\,(r+s) = -\psi^\chi + \frac{\partial Z_j}{\partial E_\phi}$$

Therefore, $\psi^\chi = \frac{\partial Z_j}{\partial E_\phi} - \gamma_\phi\,(r+s)$

**FOC wrt $N_\phi$**

$$0 = b^\chi + \gamma_\phi \frac{\partial m(N_\phi, V_\phi)}{\partial N_\phi} - \psi^\chi + \left(\mathcal{U}_\phi^U - \mu_j h_\phi^U - c_\phi^U\right)$$

plug in optimal housing consumption. Therefore, $\psi^\chi = b^\chi + \gamma_\phi \frac{\partial m(N_\phi, V_\phi)}{\partial N_\phi}$

Equating the two expressions of $\psi^\chi$ and plugging in the expression of $\frac{\partial m(N_\phi, V_\phi)}{\partial N_\phi}$, we have

$$(r + s + \alpha A N_\phi^{\alpha-1} V_\phi^{1-\alpha}) \frac{k^\chi}{(1-\alpha)AN_\phi^\alpha V_\phi^{-\alpha}} = \frac{\partial Z_j}{\partial E_\phi} - b^\chi$$

Let $\theta_\phi = \frac{V_\phi}{N_\phi}$, the expression becomes

$$\frac{k^\chi}{q(\theta_\phi)} = \frac{(1-\alpha)\frac{\partial Z_j}{\partial E_\phi} - [(1-\alpha)b^\chi + \alpha\theta_\phi k^\chi]}{r+s}$$

**For the same $\psi^\chi$**

$$b^\chi + \gamma_j^\chi \frac{\partial m(N_j^\chi, V_j^\chi)}{\partial N_j^\chi} = b^\chi + \gamma_{j'}^\chi \frac{\partial m(N_{j'}^\chi, V_{j'}^\chi)}{\partial N_{j'}^\chi}$$

Plug in the expression for $\frac{\partial m(N_\phi, V_\phi)}{\partial N_\phi} = \alpha f(\theta_\phi)$ and $\gamma_\phi$, the expression becomes

$$b^\chi + \frac{\alpha}{1-\alpha} k^\chi \theta_j^\chi = b^\chi + \frac{\alpha}{1-\alpha} k^\chi \theta_{j'}^\chi$$

which is equivalent to $\theta_j^\chi = \theta_{j'}^\chi$

**Social Planner's Solution**

Summarizing, $(N_\phi, V_\phi, E_\phi)$ would solve

$$\frac{k^\chi}{q(\theta_\phi)} = \frac{(1-\alpha)p_\phi y^\chi - [(1-\alpha)b^\chi + \alpha\theta_\phi k^\chi]}{r+s}$$
$$b^\chi + \frac{\alpha}{1-\alpha}k^\chi\theta_j^\chi = b^\chi + \frac{\alpha}{1-\alpha}k^\chi\theta_{j'}^\chi$$
$$u_\phi = \frac{s}{s+f(\theta_\phi)}$$
$$\xi = \sum_j E_j^s + N_j^s;\ 1-\xi = \sum_j E_j^n + N_j^n$$

where $\theta_\phi = V_\phi / N_\phi$.

# C Quantitative model in 6.2

## C.1 Equilbrium

### C.1.1 Bellman Equations

Let $U_\phi$, $W_\phi$, $V_\phi$, $J_\phi$ denote the value function of the unemployed, the employed, a vacant job and a filled job for each location and skill level.

$$rW_\phi(y^\chi) = w_\phi(y^\chi)R_j^{-\eta} + \lambda \int_{\underline{y}_\phi}^{\bar{y}_\phi} \max\{U_\phi - W_\phi(y^\chi), W_\phi(x^\chi) - W_\phi(y^\chi)\}dF_\chi(x^\chi) \tag{50}$$

$$rU_\phi = \max_j\{b^\chi R_j^{-\eta} + f(\theta_\phi)\int_{\underline{y}_\phi}^{\bar{y}_\phi} \max\{W_\phi(y^\chi) - U_\phi, 0\}dF_\chi(x^\chi)\} \tag{51}$$

$$rV_\phi = \max_j\{-k^\chi + q(\theta_\phi)\int_{\underline{y}_\phi}^{\bar{y}_\phi} \max\{J_\phi(x^\chi) - V_\phi, 0\}dF_\chi(x^\chi)\} \tag{52}$$

$$rJ_\phi(y^\chi) = p_\phi y^\chi - w_\phi(y^\chi) + \lambda \int_{\underline{y}_\phi}^{\bar{y^\chi}} \max\{V_\phi - J_\phi(y^\chi), J_\phi(x^\chi) - J_\phi(y^\chi)\}dF_\chi(x^\chi) \tag{53}$$

where $F_\chi(y^\chi)$ is skill distribution for skill level $\chi$.

The first Bellman equation is an employed worker's flow value. Since the worker's utility function is Cobb-Douglas, she spends $\eta$ share of her income on housing. Hence, the flow value of income is her wage adjusted by rent. The probability of matching with a firm is $f(\theta_\phi)$ for an unemployed worker. Upon meeting the firm, she draws type-specific productivity $y^\chi$ from distribution $F_\chi(.)$. At rate $\lambda$, the worker redraws productivity $x^\chi \sim F_\chi(.)$. If $x^\chi < y_\phi^*$, the match is destroyed. The worker becomes unemployed, and the firm becomes vacant. If $x^\chi \geq y_\phi^*$, the match is not destroyed and the productivity becomes $x^\chi$. The second Bellman equation is an unemployed worker's flow value. Since an unemployed worker can move between locations, the worker will choose a location that maximizes her utility. Like an employed worker, the unemployment benefit is adjusted by local rent $R_j$.

The third Bellman equation is a vacant firm's flow value. Vacant firms are also free to choose where to locate, so they will choose location j to maximize their profit. Once they settle in a location, they must pay a vacancy cost $k^\chi$. A vacant firm meets an unemployed worker at rate $q(\theta_\phi)$. The last Bellman equation is the flow value of a filled firm. The firm's profit is the value of the output less the wage paid to the worker. Similar to the Bellman equation of the employed worker, at rate $\lambda$, match productivity receives a shock $x^\chi \sim F_\chi(.)$. If $x^\chi < y^*_\phi$, the match is destroyed. The worker becomes unemployed, and the firm becomes vacant. If $x^\chi \geq y^*_\phi$, the match is not destroyed and the productivity becomes $x^\chi$.

Reservation productivity $y^*_\phi$ is chosen such that if $y^\chi < y^*_\phi$, then the job is destroyed and if $y^\chi \geq y^*_\phi$,then the match is formed keep. The Bellman equations become

$$rW_\phi(y^\chi) = w_\phi(y^\chi)R_j^{-\eta^\chi} + \lambda \int_{y^*_\phi}^{\bar{y^\chi}} [W_\phi(x^\chi) - W_\phi(y^\chi)]dF(x^\chi) - \lambda F(y^*_\phi)[W_\phi(y^\chi) - U_\phi], \tag{54}$$

$$rU_\phi = \max_j\{b^\chi R_j^{-\eta^\chi} + f(\theta_\phi) \int_{y^*_\phi}^{\bar{y^\chi}} [W_\phi(y^\chi) - U_\phi]dF(x^\chi)\}, \tag{55}$$

$$rV_\phi = -k^\chi + q(\theta_\phi) \int_{y^*_\phi}^{\bar{y^\chi}} [J_\phi(x^\chi) - V_\phi]dF(x^\chi), \tag{56}$$

$$rJ_\phi(y^\chi) = p_\phi y^\chi - w_\phi(y^\chi) + \lambda \int_{y^*_\phi}^{\bar{y^\chi}} [J_\phi(x^\chi) - J_\phi(y^\chi)]dF(x^\chi) - \lambda F(y^*_\phi)J_\phi(y^\chi). \tag{57}$$

Use $J(y^*_\phi) = 0$ and $W(y^*_\phi) = U_\phi$ to get rid of integral, yields

$$(r+\lambda)J_\phi(y^\chi) = y^\chi p_\phi - w_\phi(y^\chi) + \lambda \int_{y^*_\phi}^{\bar{y^\chi}} J_\phi(x^\chi)dF(x^\chi),$$

$$(r+\lambda)W_\phi(y^\chi) = [w_\phi(y^\chi)]R_j^{-\eta^\chi} + \lambda \int_{y^*_\phi}^{\bar{y^\chi}} W_\phi(x^\chi)dF(x^\chi) + \lambda F(y^*_\phi)U_\phi.$$

Evaluate at $y^\chi = y^*_\phi$,

$$0 = (r+\lambda)J_\phi(y^*_\phi) = p_\phi y^*_\phi - w_\phi(y^*_\phi) + \lambda \int_{y^*_\phi}^{\bar{y^\chi}} J_\phi(x^\chi)dF(x^\chi) \tag{58}$$

$$\Rightarrow (r+\lambda)J_\phi(y^\chi) = [p_\phi y^\chi - w_\phi(y^\chi_\phi)] - [p_\phi y^*_\phi - w_\phi(y^*_\phi)] \tag{59}$$

$$\Rightarrow (r+\lambda)J_\phi(y^\chi) = w_\phi(y^*_\phi) - w_\phi(y^\chi_\phi) + p_\phi[y^\chi - y^*_\phi]. \tag{60}$$

### C.1.2 Wages

Following Bilal [2023], define adjusted surplus $S_\phi$ for match productivity to be

$$S_\phi(y^\chi) = J_\phi(y^\chi) + R_j^{\eta^\chi}[W_\phi - U_\phi].$$

Nash Bargaining

$$\beta J_\phi(y^\chi) = (1-\beta)R_j^{\eta^\chi}[W_\phi(y^\chi) - U_\phi], \quad \forall y^\chi \geq y^*_\phi$$

$$\Rightarrow \beta \int_{y^*_\phi}^{\bar{y^\chi}} J_\phi(x^\chi)dF_\phi(x^\chi) = (1-\beta) \int_{y^*_\phi}^{\bar{y^\chi}} [W_\phi(x^\chi) - U_\phi]dF_\phi(x^\chi).$$

With free entry condition, $V_\phi = 0$ and equation (56),

$$\frac{k^\chi}{q(\theta_\phi)} = \int_{y^*_\phi}^{\bar{y^\chi}} J_\phi(x^\chi)dF(x^\chi).$$

Plug the expression of $J_\phi$ into Nash bargaining rule to get expression of $\int_{y^*_\phi}^{\bar{y^\chi}}[W_\phi(y^\chi) - U_\phi]dF(x^\chi)$,

$$\int_{y^*_\phi}^{\bar{y^\chi}} [W_\phi(y^\chi) - U_\phi]dF(x^\chi) = \frac{\beta}{1-\beta}\frac{k^\chi}{q(\theta_\phi)}R_j^{-\eta^\chi}.$$

Plug this expression into the Bellman equation for the unemployed $U_\phi$,

$$rU_\phi = \max_j \{b^\chi R_j^{-\eta^\chi} + f(\theta_\phi)\int_{y_\phi^*}^{\bar{y^\chi}} [W_\phi(y^\chi) - U_\phi]dF(x^\chi)\},$$

$$\Rightarrow rU_\phi = \max_j (b^\chi + \frac{\beta}{1-\beta}k^\chi\theta_\phi)R_j^{-\eta}.$$

With spatial equilibrium $U_j^\chi = U_{j'}^\chi = \bar{U}^\chi, \ \ \forall j$, the Bellman equation for $U_\phi$ becomes

$$r\bar{U}^\chi = (b^\chi + \frac{\beta}{1-\beta}k^\chi\theta_\phi)R_j^{-\eta}. \tag{61}$$

Subtract Bellman equations and re-arrange

$$\begin{aligned} r[W_\phi(y^\chi) - \bar{U}^\chi] &= R_j^{-\eta^\chi}[w_\phi(y^\chi) - b^\chi - \frac{\beta}{1-\beta}\theta_\phi k^\chi] + \lambda \int_{y_\phi^*}^{\bar{y^\chi}} [W_\phi(x^\chi) - U_\phi]dF(x^\chi) \\ &- \lambda \int_{y_\phi^*}^{\bar{y^\chi}} [W_\phi(y^\chi) - U_\phi]dF(y^\chi) - \lambda F(y_\phi^*)[W_\phi(y^\chi) - U_\phi], \\ rJ_\phi(y^\chi) &= p_\phi y^\chi - w_\phi(y^\chi) + \lambda \int_{y_\phi^*}^{\bar{y^\chi}} J_\phi(x^\chi)dF(x^\chi) \\ &- \lambda \int_{y_\phi^*}^{\bar{y^\chi}} J_\phi(y^\chi)dF(x^\chi) - \lambda F(y_\phi^*)J_\phi(y^\chi). \end{aligned}$$

Use Nash Bargaining $(1-\beta)R_j^{\eta^\chi}(W_\phi - U_\phi) = \beta(J_\phi - V)$,

$$\begin{aligned} \beta[p_\phi y^\chi - w_\phi(y^\chi)] &= R_j^{\eta^\chi}(1-\beta)\{R_j^{-\eta^\chi}[w_\phi(y^\chi) - b^\chi - \frac{\beta}{1-\beta}\theta_\phi k^\chi]\} \\ \Rightarrow \beta[p_\phi y^\chi - w_\phi(y^\chi)] &= (1-\beta)[w_\phi(y^\chi) - b^\chi - \frac{\beta}{1-\beta}\theta_\phi k^\chi] \\ \Rightarrow w_\phi(y^\chi) &= \beta p_\phi y^\chi + (1-\beta)b^\chi + \beta\theta_\phi k^\chi. \end{aligned}$$

**Job Creation Condition**

Evaluate $w_\phi(y^\chi)$ at $y^\chi = y_\phi^*$ and subtract it from $w_\phi(y^\chi)$ yields,

$$w_\phi(y^\chi) - w_\phi(y_\phi^*) = \beta p_\phi(y^\chi - y_\phi^*).$$

Plug the expression into equation(60)

$$(r+\lambda)J_\phi(y^\chi) = w_\phi(y_\phi^*) - w_\phi(y_\phi^\chi) + p_\phi[y^\chi - y_\phi^*]$$

$$\Rightarrow (r+\lambda)J_\phi(y^\chi) = (y^\chi - y_\phi^*)p_\phi(1-\beta)$$

Re-arrange equation (56),

$$\begin{aligned} rV_\phi &= -k^\chi + q(\theta_\phi)\int_{y_\phi^*}^{\bar{y^\chi}} [J_\phi(x^\chi) - V_\phi]dF(x^\chi) \\ \Rightarrow k^\chi &= q(\theta_\phi)(1 - F(y_\chi^*))\int_{y_\phi^*}^{\bar{y^\chi}} [J_\phi(x^\chi) - V_\phi]\frac{dF(x^\chi)}{1 - F(y_\chi^*)} \\ \Rightarrow k^\chi &= q(\theta_\phi)(1 - F(y_\phi^*))[J_\phi^e - V_\phi]. \end{aligned}$$

where $J_\phi^e = E[J_\phi(y^\chi)|y^\chi \geq y_\phi^*]$. Therefore, job creation condition is

$$\frac{k^\chi}{q(\theta_\phi)[1 - F(y_\phi^*)]} = \frac{p_\phi(1-\beta)(y_\phi^e - y_\phi^*)}{r+\lambda}, \tag{62}$$

where $y_\phi^e = E[y_\phi | y_\phi \geq y_\phi^*]$.

**Job Destruction Condition**

Plug $w$ into the Bellman equation of $J_\phi$

$$(r+\lambda)J_\phi(y^\chi) = p_\phi y^\chi - (\beta p_\phi y^\chi + [(1-\beta)b + \beta\theta_\phi k^\chi]) + \lambda \int_{y_\phi^*}^{\bar{y^\chi}} J_\phi(x^\chi)dF(x^\chi). \tag{63}$$

Evaluate at $y^\chi = y_\phi^*$ and subtracting the resulting equation from equation (63),

$$(r+\lambda)J_\phi(y^\chi) = (1-\beta)p_\phi(y^\chi - y_\phi^*).$$

Plug this expression into $J$ of equation (63),

$$(r+\lambda)J_\phi(y^\chi) = (1-\beta)p_\phi y^\chi - [(1-\beta)b^\chi + \beta\theta_\phi k^\chi] + (1-\beta)\frac{p_\phi\lambda}{r+\lambda}\int_{y_\phi^*}^{\bar{y^\chi}} (y^\chi - y_\phi^*)dF(x^\chi).$$

Evaluate this equation at $y^\chi = y_\phi^*$, and use $J_\phi(y_\phi^*) = 0$ to get the Job Destruction Condition,

$$p_\phi y_\phi^* - [b^\chi + \frac{\beta}{1-\beta}\theta_\phi k^\chi] + \frac{p_\phi\lambda}{r+\lambda}\int_{y_\phi^*}^{\bar{y^\chi}} (y^\chi - y_\phi^*)dF(x^\chi) = 0. \tag{64}$$

### C.1.3 Equilibrium Conditions

Job Creation condition (62) and Job Destruction Condition (64) determine equilibrium $(\theta_\phi^*, y_\phi^{**})$ for each $\phi$. JC: As $\theta_\phi \uparrow \Rightarrow q(\theta_\phi) \downarrow \Rightarrow y_\phi^* \downarrow$. JD: As $\theta_\phi \uparrow \Rightarrow y_\phi^* \uparrow$

**Beverage Curve**

$$u_\phi = \frac{\lambda F(y_\phi^*)}{\lambda F(y_\phi^*) + f(\theta_\phi)}. \tag{65}$$

The shape of the distribution $F(y_\phi^*)$ affects the unemployment rate and hence the job finding rate. For the same reservation productivity $y_\phi^*$, the fatter the tail of $F(y_\phi^*)$, the smaller the value of $F(y_\phi^*)$.

**Spatial Equilibrium Condition**

$$(\frac{R_j}{R_{j'}})^{-\eta} = \frac{(b^\chi + \frac{\beta}{1-\beta}k^\chi\theta_{j'}^\chi)}{(b^\chi + \frac{\beta}{1-\beta}k^\chi\theta_j^\chi)}, \tag{66}$$

therefore, the difference in housing price between the two locations is explained by the difference $\theta_\phi\mu_j$.

**Market clearing condition for housing**

$$R_j = \frac{\eta\{L_j^n[\bar{w_j^n}(1-u_j^n) + b^n u_j^n] + L_j^s[\bar{w_j^s}(1-u_j^s) + b^s u_j^s]\}}{Q_j}.$$

**Market clearing condition for workers**

$$\sum_j L_j = 1; \quad \xi = \sum_j \zeta_j L_j.$$

**Equilibrium Equations**

$$0 = p_\phi y^*_\phi - [b^\chi + \frac{\beta}{1-\beta}\theta_\phi k^\chi] + \frac{p_\phi \lambda}{r+\lambda}\int_{y^*_\phi}^{\bar{y^\chi}} [y^\chi - y^*_\phi] dF(x^\chi) \tag{67}$$

$$\frac{k^\chi}{q(\theta_\phi)[1-F(y^*_\chi)]} = \frac{p_\phi(1-\beta)[y^e_\phi - y^*_\phi]}{r+\lambda} \tag{68}$$

$$u_\phi = \frac{\lambda F(y^*_\phi)}{\lambda F(y^*_\phi) + f(\theta_\phi)} \tag{69}$$

$$R_j = \frac{\eta\{L^n_j[\bar{w^n_j}(1-u^n_j) + b^n u^n_j] + L^s_j[\bar{w^s_j}(1-u^s_j) + b^s u^s_j]\}}{Q_j} \tag{70}$$

$$(\frac{R_j}{R_{j'}})^{-\eta} = \frac{(b^\chi + \frac{\beta}{1-\beta}k^\chi \theta^\chi_{j'})}{(b^\chi + \frac{\beta}{1-\beta}k^\chi \theta^\chi_j)} \tag{71}$$

$$w_\phi(y^\chi) = \beta p_\phi y^\chi + (1-\beta)b^\chi + \beta\theta_\phi k^\chi \tag{72}$$

$$\sum_j L_j = 1; \qquad \xi = \sum_j L_j \zeta_j \tag{73}$$

### C.1.4 Defitinition of equilibrium

**Definition 3.** *A steady-state equilibrium is* $\{w_\phi, y^*_\phi, u_\phi, \theta_\phi, p_\phi, \zeta_j, L_j, R_j\}$ *for* $\phi \in J \times \{s, n\}$ *and* $j \in J$ *such that: equations (2)- (1), (5)-(6),(11),(62),(64),(65),(66) are satisfied*

## C.2 Planner's Problem

The Social Planner's problem is very similar to the baseline version presented in Section 5. The derivation for the social planner's solution is summarized here. The social planner aims to maximize a social welfare function subject to resource constraints and the law of motion of unemployment. The social welfare function assigns equal welfare weights for the three groups of agents: two types of workers and absentee landlords. Let $N_\phi$ denote the number of unemployed workers of type $\phi$, and let $E_\phi$ denote the number of employed workers of type $\phi$.

The planner's objective function is

$$\omega = \int_0^\infty e^{-rt}\left(\sum_\phi \left[\left(\frac{c^E_\phi}{1-\eta}\right)^{1-\eta}\left(\frac{h^E_\phi}{\eta}\right)^\eta \times E_\phi + \left(\frac{c^U_\phi}{1-\eta}\right)^{1-\eta}\left(\frac{h^U_\phi}{\eta}\right)^\eta \times N_\phi\right] + \sum_j c^O_j\right) dt,$$

where the first component is the aggregate utility of the employed workers, the second component is the aggregate utility of the unemployed workers, and the last component is the consumption of out-of-town landlords.

The planner picks market tightness ($\theta_\phi$) reservation productivity $y^*_\phi$ and labor force size ($L_\phi$) for each $\phi$, as well as housing and non-housing consumption for workers and landlord ($c^E_\phi, c^U_\phi, h^E_\phi, h^U_\phi, c^O_j$). The constraints the planner faces are (1) the law of motion for unemployment (for each $\phi$), (2) land clearing for each location (for each j), (3) resource constraint of the planner, (4) high-skilled worker size and population constraints. The current-value Hamiltonian for the planner is

$$H = \sum_j [(\frac{c_j^{sE}}{1-\eta})^{1-\eta}(\frac{h_j^{sE}}{\eta})^{\eta}\zeta_j(1-u_j^s) + (\frac{c_j^{sU}}{1-\eta})^{1-\eta}(\frac{h_j^{sU}}{\eta})^{\eta}\zeta_j(1-u_j^s) + (\frac{c_j^{nE}}{1-\eta})^{1-\eta}(\frac{h_j^{nE}}{\eta})^{\eta}(1-\zeta_j)(1-u_j^n)$$

$$+(\frac{c_j^{nU}}{1-\eta})^{1-\eta}(\frac{h_j^{nU}}{\eta})^{\eta}u_j^n(1-\zeta_j)]L_j + \sum_\phi \gamma_\phi \left[A\theta_\phi^{1-\alpha}u_\phi - \lambda F(y_\phi^*)(1-u_\phi)\right]$$

$$+\sum_j \{[y_j^{es}(1-u_j^s)\zeta_j]^{\sigma_j}[y_j^{en}(1-u_j^n)(1-\zeta_j)]^{1-\sigma_j} - c_j^{sE}\zeta_j(1-u_j^s) - c_j^{nE}(1-\zeta_j)(1-u_j^n)$$

$$+(b^s - k^s\theta_j^s - c_j^{sU})\zeta_j u_j^s + (b^n - k^n\theta_j^n - c_j^{nU})(1-\zeta_j)u_j^n\}L_j + \psi^s\left[\xi - \sum_j L_j^s\right] + \psi^n\left[1-\xi-\sum_j L_j^n\right]$$

$$+\sum_j \kappa_j\left(Q_j - L_j\left[h_j^{sE}\zeta_j(1-u_j^s) + h_j^{sU}\zeta_j u_j^s + h_j^{nE}(1-\zeta_j)(1-u_j^n) + h_j^{nU}(1-\zeta_j)u_j^n\right]\right)$$

**Optimal consumption and housing**

First order conditions wrt $(h_\phi, c_\phi)$

$$\frac{\partial H}{\partial c_\phi^E} = 0,\ \frac{\partial H}{\partial c_\phi^U} = 0 \Rightarrow 1 = \frac{1-\eta}{c_\phi^E}\mathcal{U}_\phi^E = \frac{1-\eta}{c_\phi^U}\mathcal{U}_\phi^U$$
$$\frac{\partial H}{\partial h_\phi^U} = 0,\ \frac{\partial H}{\partial h_\phi^U} = 0 \Rightarrow \kappa_j = \frac{\eta}{h_\phi^E}\mathcal{U}_\phi^E = \frac{\eta}{h_\phi^U}\mathcal{U}_\phi^U$$

The first two FOCs lead to the following equation

$$h_\phi^E = \frac{c_\phi^E}{\kappa_j}\frac{\eta}{1-\eta};\quad h_\phi^U = \frac{c_\phi^U}{\kappa_j}\frac{\eta}{1-\eta};\quad \mathcal{U}_\phi^E = \frac{c_\phi^E}{1-\eta};\quad \mathcal{U}_\phi^U = \frac{c_\phi^U}{1-\eta}$$

**Planner's FOC wrt** $(\theta_\phi)$

$$\frac{\partial H}{\partial \theta_j^s} = 0 \Rightarrow -u_j^s\zeta_j L_\phi \mu_j k^\chi + \gamma_\phi(1-\alpha)A(\theta_j^s)^{-\alpha}u_\phi = 0 \Rightarrow \gamma_j^s = \frac{k^s\zeta_j L_j}{(1-\alpha)A\theta_\phi^{-\alpha}}$$
$$\frac{\partial H}{\partial \theta_j^n} = 0 \Rightarrow -u_j^s(1-\zeta_j)L_j\mu_j k^\chi + \gamma_\phi(1-\alpha)A(\theta_j^n)^{-\alpha}u_\phi = 0 \Rightarrow \gamma_j^n = \frac{k^n(1-\zeta_j)L_j}{(1-\alpha)A\theta_\phi^{-\alpha}}$$

**Planner's FOC wrt** $(y_\phi^*)$

$$\frac{\partial H}{\partial y_\phi^*} = 0 \Rightarrow p_\phi(1-u_\phi)L_\phi\frac{\partial y^e}{\partial y^*} - \gamma_\phi\lambda(1-u_\phi)\frac{\partial F(y_\phi^*)}{\partial y_\phi^*} = 0$$

Note that,

$$\frac{\partial y^e}{\partial y_\phi^*} = \frac{\partial}{\partial y_\phi^*}\left([1-F(y_\phi^*)]^{-1}\int_{y_\phi^*} y_\phi dF(y_\phi)\right)$$
$$= f(y_\phi^*)[1-F(y_\phi^*)]^{-2}\int_{y_\phi^*} y_\phi dF(y_\phi) + [1-F(y_\phi^*)]^{-1}(-y_\phi^* f(y_\phi^*))$$
$$= f(y_\phi^*)[1-F(y_\phi^*)]^{-2}[1-F(y_\phi^*)]y_\phi^e - [1-F(y_\phi^*)]^{-1}y_\phi^* f(y_\phi^*)$$
$$= f(y_\phi^*)[1-F(y_\phi^*)]^{-1}(y_\phi^e - y_\phi^*)$$

$$\Rightarrow p_\phi(1-u_\phi)L_\phi f(y^*_\phi)[1-F(y^*_\phi)]^{-1}(y^e_\phi - y^*_\phi) - \gamma_\phi \lambda(1-u_\phi)f(y^*_\phi) = 0$$

Plug in $\gamma_\phi$

$$\frac{k^\chi}{[1-F(y^*)]q(\theta_\phi)} = \frac{(1-\alpha)(y^e_\phi - y^*_\phi)}{r+\lambda}$$

**Planner's FOC wrt** $(L_\phi)$

$$\frac{\partial H}{\partial L_\phi} = 0 \Rightarrow \psi^\chi = \frac{\partial Z_j}{\partial L^\chi_j} + (b^\chi_j - k^\chi \theta^\chi_j)u^\chi_j.$$

Plug in the expression for $p_\phi$

$$p^s_j = \sigma_j \left(Y^s_j\right)^{\rho-1} Z_j^{1-\rho}; \quad p^n_j = (1-\sigma_j)\left(Y^n_j\right)^{\rho-1} Z_j^{1-\rho}.$$

Therefore, the spatial optimality condition is

$$p^\chi_j y^{e,\chi}_j (1-u^\chi_j) + (b^\chi_j - k^\chi \theta^\chi_j)u^\chi_j = p^\chi_{j'} y^{e,\chi}_{j'}(1-u^\chi_{j'}) + (b^\chi_{j'} - k^\chi \theta^\chi_{j'})u^\chi_{j'}, \quad \forall \chi.$$

**Equation for co-state variable** $u_\phi$

$$\frac{\partial H}{\partial u_\phi} = r\gamma_\phi - \dot{\gamma}_\phi \Rightarrow r\gamma_\phi - \dot{\gamma}_\phi = -\gamma_\phi[A(\theta_\phi)^{1-\alpha} + \lambda F(y^*_\phi) + s] + \frac{\partial Z_j}{\partial u_\phi} + L_\phi(b^\chi - k^\chi \theta_\phi)$$

Plug in $\gamma_\phi$ and impose steady state condition $\dot{\gamma_\phi} = 0$ and re-arrange,

$$0 = p_\phi y^*_\phi - [b_\phi + \frac{\alpha}{1-\alpha}\theta_\phi k^\chi] + \frac{p_\phi \lambda}{r+\lambda}\int_{y^*_\phi}^{\bar{y^\chi}} (y^\chi - y^*_\phi) dF(x^\chi).$$

Planner's optimal choice of $\{\theta_\phi, y^*_\phi, L_\phi\}, \forall \phi$ will satisfy the following conditions

$$0 = p_\phi y^*_\phi - [b_\phi + \frac{\alpha}{1-\alpha}\theta_\phi k^\chi] + \frac{p_\phi \lambda}{r+\lambda}\int_{y^*_\phi}^{\bar{y^\chi}} (y^\chi - y^*_\phi) dF(x^\chi),$$

$$\frac{k^\chi}{[1-F(y^*)]q(\theta_\phi)} = \frac{(1-\alpha)(y^e_\phi - y^*_\phi)}{[1-F(y^*)](r+\lambda)},$$

$$p^\chi_j y^{e,\chi}_j (1-u^\chi_j) + (b^\chi_j - k_j \theta^\chi_j)u^\chi_j = p^\chi_{j'} y^{e,\chi}_{j'}(1-u^\chi_{j'}) + (b^\chi_{j'} - k^\chi \theta^\chi_{j'})u^\chi_{j'},$$

$$u_\phi = \frac{\lambda F(y^*_\phi)}{\lambda F(y^*_\phi) + f(\theta_\phi)},$$

$$1 = \sum_j L_j;\ \xi = \sum_j L_j \zeta_j$$

# D Data

## D.1 Occupation-based skill definition

Using the AM measure, the occupation with the highest and lowest skills would be

Table 4: Occupation with the highest and lowest AM

| Highest 20 | AM | Lowest 20 | AM |
|---|---|---|---|
| Physical Scientist | 1 | Dancers | 0 |
| Chemical Engineers | 0.983 | Parking Lot Attendant | 0.222 |
| Chemists | 0.952 | Paving, surfacing, and tamping equipment operators | 0.253 |
| Actuaries | 0.944 | Operating Engineers of construction equipment | 0.273 |
| Dietitians and Nutritionists | 0.942 | Fire Fighting | 0.273 |
| Metallurgical and Materials Engineers | 0.926 | Excavating and Loading Machine Operators | 0.281 |
| Mechanical Engineers | 0.926 | Bus Driver | 0.283 |
| Funeral Directors | 0.924 | Truck, Delivery, and Tractor Drivers | 0.283 |
| Accountants and Auditors | 0.922 | Taxi Cab Driver | 0.285 |
| Petroleum, Mining and Geological Engineers | 0.921 | Roofer and Slaters | 0.291 |
| Managers of Medicine | 0.914 | Crane, derrick, winch, and hoist operators | 0.291 |
| Financial Managers | 0.911 | Structural Metal Workers | 0.302 |
| Aerospace Engineer | 0.897 | Plasterers | 0.306 |
| Atmospheric and Space Scientists | 0.895 | Textile and Sewing Machine Operator | 0.343 |
| Other Financial Specialist | 0.893 | Garbage and Recyclable Material Collector | 0.343 |
| Subject Instructor (HS/College) | 0.892 | Driller of Earth | 0.361 |
| Managers and Specialists in Marketing, Advertising, and Public relations | 0.883 | Railroad brake, coupler, and switch operators | 0.362 |
| Biological Scientists | 0.882 | Millwrights | 0.370 |
| Computer Software Developer | 0.879 | Carpenter | 0.371 |

# E Policy Experiment Equilibrium

## E.1 Policy Experiments

### E.1.1 Relocation subsidies

A relocation subsidy $\tau^m$ for low-skill workers in location H. The subsidies are financed by lump-sum tax $\tau^c$ on workers, regardless of employment status. The size of the subsidy equals 10 percent of housing spending an unemployed low-skill worker in location H would pay.

The Bellman equations become

$$rW_\phi(y^\chi) = [\mathbf{1}_{j=H,\chi=n}\tau^m - \tau^c + w_\phi(y^\chi)]R_j^{-\eta^\chi} + \lambda \int_{y_\phi^*}^{\bar{y^\chi}} [W_\phi(x^\chi) - W_\phi(y^\chi)]dF(x^\chi) - \lambda F(y_\phi^*)[W_\phi(y^\chi) - U_\phi], \tag{74}$$

$$rU_\phi = \max_j \left\{ [\mathbf{1}_{j=H,\chi=n}\tau^m - \tau^c + b_\phi]R_j^{-\eta^\chi} + f(\theta_\phi) \int_{y_\phi^*}^{\bar{y^\chi}} [W_\phi(y^\chi) - U_\phi]dF(x^\chi) \right\}. \tag{75}$$

$$rV_\phi = -k^\chi + q(\theta_\phi) \int_{y_\phi^*}^{\bar{y^\chi}} [J_\phi(x^\chi) - V_\phi]dF(x^\chi), \tag{76}$$

$$rJ_\phi(y^\chi) = p_\phi y^\chi - w_\phi(y^\chi) + \lambda \int_{y_\phi^*}^{\bar{y^\chi}} [J_\phi(x^\chi) - J_\phi(y^\chi)]dF(x^\chi) - \lambda F(y_\phi^*)J_\phi(y^\chi), \tag{77}$$

where $\mathbf{1}_{j=H,\chi=n}$ is an indicator function that equals to 1 if location $j = H$ and skill type $\chi = n$, and equals to 0 otherwise. The wage equation becomes

$$w_\phi(y^\chi) = \beta p_\phi y^\chi + [(1-\beta)b_\phi + \beta\theta_\phi k^\chi] \tag{78}$$

The equilibrium conditions with policy instruments are the following

$$r\bar{U}^{\chi} = [\mathbf{1}_{j=H,\chi=n}\tau^{m} - \tau^{c} + b_{\phi} + \frac{\beta}{1-\beta}(k^{\chi} + \tau^{c})\theta_{\phi}]R_j^{-\eta}, \tag{79}$$

$$\frac{k^{\chi}}{q(\theta_{\phi})[1 - F(y_{\chi}^{*})]} = \frac{p_{\phi}(1-\beta)(y_{\phi}^{e} - y_{\phi}^{*})}{r+\lambda}. \tag{80}$$

$$0 = p_{\phi}y_{\phi}^{*} - [b_{\phi} + \frac{\beta}{1-\beta}\theta_{\phi}k^{\chi}] + \frac{p_{\phi}\lambda}{r+\lambda}\int_{y_{\phi}^{*}}^{\bar{y^{\chi}}}[y^{\chi} - y_{\phi}^{*}]dF(x^{\chi}), \tag{81}$$

$$u_{\phi} = \frac{\lambda F\left(y_{\phi}^{*}\right)}{\lambda F\left(y_{\phi}^{*}\right) + f\left(\theta_{\phi}\right)}. \tag{82}$$

The subsidies for the workers are financed by a lump-sum tax $\tau^{c}$ on workers, regardless of employment status, skill, or location. The subsidy is given to the workers such that the size of housing consumption is

$$t^{m} = 0.1 \times b_{H}^{n}\eta$$
$$t^{c} = t^{m}[(L_H(1-\zeta_H)]$$

# F Proofs and Discussions

## F.1 Proof of Proposition 1

By the spatial equilibrium condition, reproduced here for convenience,

$$\overline{U}^{\chi} = \left(b^{\chi} + \frac{\beta}{1-\beta}k^{\chi}\theta_j^{\chi}\right)R_j^{-\eta}, \tag{83}$$

we can see that within each skill type, workers are indifferent between locations. If one location's market tightness is higher, i.e., $\theta_j^{\chi} > \theta_{j'}^{\chi}$, then $R_j > R_{j'}$ must be true to maintain the spatial equilibrium condition since the rest of the elements in the equations do not vary by location. Additionally, the Beverage Curve dictates a negative relationship between market tightness and unemployment rate, i.e., if $\theta_j^{\chi} > \theta_{j'}^{\chi}$, then $u_j^{\chi} < u_{j'}^{\chi}$. Combining these two inequalities, we can see that if the location with a higher rent also features a lower unemployment rate for each skill type, i.e., if $R_j > R_{j'}$, then $u_j^{\chi} < u_{j'}^{\chi}$.

## F.2 Proof of Corollary 1

The job creation condition, reproduced here for convenience,

$$\frac{k^{\chi}}{q(\theta_{\phi})} = \frac{(1-\beta)p_{\phi}y^{\chi} - [(1-\beta)b^{\chi} + \beta\theta_{\phi}k^{\chi}]}{r+s},$$

which shows that when market tightness $\theta_{\phi}$ increases, the price of the intermediate goods $p_{\phi}$ must also increase. Therefore, within each skill type $\chi$, if the market tightness is bigger in one location, then the intermediate goods' price must be higher in that location, i.e., if $\theta_j^{\chi} > \theta_k^{\chi}$, then $p_j^{\chi} > p_k^{\chi}$. The wage equation, reproduced here for convenience,

$$w_{\phi} = \beta p_{\phi}y^{\chi} + [(1-\beta)b^{\chi} + \beta\theta_{\phi}k^{\chi}],$$

shows that wage increases in both the market tightness and the intermediate goods' price. Since we already know that $p_\phi$ also increases with $\theta_\phi$, we can say that if $\theta_j^\chi > \theta_k^\chi$, then $w_j^\chi > w_k^\chi$. By the Beveridge Curve (24), we know that the unemployment rate is decreasing in market tightness, therefore if $u_j^\chi < u_{j'}^\chi$, then $w_j^\chi > w_{j'}^\chi$ $\forall j, j' \in J$ and $\chi \in \{s, n\}$

## F.3 Proof of Corollary 2

The real wage's expression is $\tilde{w}_j^\chi = \frac{w_j^\chi}{R_j^\eta}$. Plug the spatial equilibrium into the wage equation, and then plug in the job creation condition

$$w_\phi = \beta p_\phi y^\chi + [(1-\beta)b^\chi + \beta\theta_\phi k^\chi] \tag{84}$$

$$= \beta p_\phi y^\chi + \frac{1}{1-\beta}\overline{U}^\chi R_j^\eta \tag{85}$$

$$\Rightarrow \overline{U}^\chi = \tilde{w}_j^\chi - \frac{\beta}{1-\beta}R_j^{-\eta}\left[(r+s^\chi)\frac{k^\chi}{q(\theta_j^\chi)} + (1-\beta)b^\chi + \beta\theta_j^\chi k^\chi\right] \tag{86}$$

Since $\overline{U}^\chi$ does not vary across space, for both $\tilde{w}_j^\chi > \tilde{w}_{j'}^\chi$ and $\theta_j > \theta_{j'}$ to be satisfied, it must be true that $\frac{p_j^\chi y^\chi}{R_j^\eta} > \frac{p_{j'}^\chi y^\chi}{R_{j'}^\eta}$. By the Beveridge Curve (24), we know that the unemployment rate is decreasing in market tightness; therefore, $u_j^\chi < u_{j'}^\chi$, and $\tilde{w}_j^\chi > \tilde{w}_{j'}^\chi$ when $\frac{p_j^\chi y^\chi}{R_j^\eta} > \frac{p_{j'}^\chi y^\chi}{R_{j'}^\eta}$. The theoretical relationship between real wages and unemployment rates is less conclusive.

## F.4 Proof of proposition 2

1. Case 1: $\sigma_j = \sigma_k$, $Q_j > T_k$. We can implement $\uparrow \frac{\sigma_j}{\sigma_k}$ by raising $\sigma_j$ while holding $\sigma_k$ constant. In this case, the production side is symmetrical i.e. $p_j^\chi = p_k^\chi$, $\zeta_j = \zeta_k$, but the housing market side is different. Since more land is available in location j, from the Spatial Equilibrium equation (23), it must be that $R_j = R_k$. By housing cost equation (11), since $R_j = R_k$, $\zeta_j = \zeta_k, w_j^\chi = w_k^\chi, u_j^\chi = u_k^\chi$, hence it must be that $L_j > L_k$ to balance the difference in $Q_j > T_k$. All the other variables have the same value for each $\phi$.

2. Case 2: $\sigma_j > \sigma_k$, $Q_j = T_k$. We can implement $\uparrow \frac{Q_j}{T_k}$ by raising $Q_j$ while holding $T_k$ constant. From spatial equilibrium for high-skill worker

$$\left(b^s + \frac{\beta}{1-\beta}k^s\theta_j^s\right)R_j^{-\eta} = \left(b^s + \frac{\beta}{1-\beta}k^s\theta_k^s\right)R_k^{-\eta}.$$

From spatial equilibrium for low-skill worker

$$\left(b^n + \frac{\beta}{1-\beta}k^n\theta_j^n\right)R_j^{-\eta} = \left(b^n + \frac{\beta}{1-\beta}k^n\theta_k^n\right)R_k^{-\eta}$$

.

Hence

$$\frac{b^s + \frac{\beta}{1-\beta}k^s\theta_j^s}{b^s + \frac{\beta}{1-\beta}k^s\theta_k^s} = \frac{b^n + \frac{\beta}{1-\beta}k^n\theta_j^n}{b^n + \frac{\beta}{1-\beta}k^n\theta_k^n}.$$

Since $b^s = b^n, k^s = k^n$, then it must be that $\theta_j^s = \theta_k^s$ and $\theta_j^n = \theta_k^n$, therefore $u_j^s = u_k^s, u_j^n = u_k^n, p_j^s = p_k^s, p_j^n = p_k^n$. Going back to the spatial equilibrium condition, $R_j = R_k$.

Since $p_j^s = p_k^s$ and $\sigma_j > \sigma_k$, using the price equation 2 and 1, it must be that the $\zeta_j > \zeta_k$.

Since $p_j^s = p_k^s$, the ratio between $p_j^s$ and $p_k^s$ is

$$1 = \frac{\sigma_j \rho_j}{\sigma_k \rho_k} \left( \frac{1 - u_j^s}{1 - u_j^n} \right)^{\sigma_j \rho_j - \sigma_j \rho_k} \frac{\left( \frac{\zeta_j}{1-\zeta_j} \right)^{\sigma_j \rho_j - 1}}{\left( \frac{\zeta_k}{1-\zeta_k} \right)^{\sigma_k \rho_k - 1}}.$$

Since $\sigma_j > \sigma_k$ and $\zeta_j > \zeta_k$, it must be that $u_j^s < u_j^n$, and hence due to the Beveridge Curve, it must be true that $\theta_j^s > \theta_j^n$.

## F.5 Proof of proposition 3

Since the location-dependent parameters are symmetrical, the production functions are the same across locations, and so are the housing supplies. Therefore, the skill composition and total worker size will also be symmetrical across locations. and $L_j / L_{j'}$ and $\zeta_j / \zeta_{j'}$ will not change even if skill dependent parameter changes.

1. Case 1: $y^s > y^n, b^s = b^n, k^s = k^n$ We can implement $\uparrow \frac{y^s}{y^n}$ by raising $y^s$ while holding $y^n$ constant. As $y^s$ increases, surpluses for both types of matches increase. Therefore, market tightness increases for all $\phi$, and unemployment rates decrease for all $\phi$. Since $b^s = b^n, k^n = k^s$, the ratio for $\theta_j^s / \theta_j^n$ and $u_j^s / u_j^n$ stay the same.

2. Case 2: $y^s = y^n, b^s > b^n, k^s = k^n$. We can implement $\uparrow \frac{b^s}{b^n}$ by raising $b^s$ while holding $b^n$ constant. By the job creation condition, as $b^s$ increases, $\theta_j^s$ decreases. Since $\theta_j^n$ stays the same, $\frac{\theta_j^s}{\theta_j^n}$ decreases. By the Beveridge Curve, we can see that when $\theta_\phi$ increases, $u_\phi$ decreases. Therefore, $\frac{u^s}{u^n}$ increases as $\frac{b^s}{b^n}$ decreases.

3. Case 3: $y^s = y^n, b^s = b^n, k^s > k^n$. We can implement $\uparrow \frac{k^s}{k^n}$ by raising $k^s$ while holding $k^n$ constant. By the job creation condition, as $k^s$ increases, $\theta_j^s$ decreases. Since $\theta_j^n$ stays the same, $\frac{\theta_j^s}{\theta_j^n}$ decreases. By the Beveridge Curve, we can see that when $\theta_\phi$ increases, $u_\phi$ decreases. Therefore, $\frac{u^s}{u^n}$ increases as $\frac{k^s}{k^n}$ decreases.

By the spatial equilibrium condition, reproduced here for convenience,

$$\overline{U}^\chi = \left( b^\chi + \frac{\beta}{1-\beta} k^\chi \theta_j^\chi \right) R_j^{-\eta}, \tag{87}$$

we can see that within each skill type, workers are indifferent between locations. If one location's market tightness is higher, i.e., $\theta_j^\chi > \theta_{j'}^\chi$, then $R_j > R_{j'}$ must be true to maintain the spatial equilibrium condition since the rest of the elements in the equations do not vary by location.

The Beverage Curve in the extended model has a different expression than in the baseline model, reproduced here for convenience,

$$u_\phi = \frac{\lambda F(y_\phi^*)}{\lambda F(y_\phi^*) + f(\theta_\phi)}.$$

Since we already know that if $R_j > R_{j'}$, then $\theta_j^\chi > \theta_{j'}^\chi$, in order for the result from Proposition 1 to hold, we need to show that

Suppose $\theta_j^{\chi} = \theta_{j'}^{\chi} + \Delta$

Additionally, the Beverage Curve dictates a negative relationship between market tightness and unemployment rate, i.e., if $\theta_j^{\chi} > \theta_{j'}^{\chi}$, then $u_j^{\chi} < u_{j'}^{\chi}$. Combining these two inequalities, we can see that if the location with a higher rent also features a lower unemployment rate for each skill type, i.e., if $R_j > R_{j'}$, then $u_j^{\chi} < u_{j'}^{\chi}$.

## F.6 Proof of Proposition 4

Subtract the wage difference in the frictional labor market (Equation 31) from the wage difference in the competitive labor market (Equation 30).

$$\Delta\breve{w}^{\chi} - \Delta w^{\chi} = \Delta\breve{p}^{\chi} y^{\chi} - [\beta y^{\chi}\Delta p^{\chi} + \beta k^{\chi}\Delta\theta^{\chi}]$$

Therefore, if $\Delta\breve{p}^{\chi} y^{\chi} - [\beta y^{\chi}\Delta p^{\chi} + \beta k^{\chi}\Delta\theta^{\chi}] > 0$, then $\Delta\breve{w}^{\chi} > \Delta w^{\chi}$, the location wage gap is bigger in the competitive labor market than the frictional labor market; Otherwise, $\Delta\breve{w}^{\chi} < \Delta w^{\chi}$, the location wage gap is bigger in the frictional labor market than in the competitive labor market.

## F.7 Proof of Proposition 5

The first condition,

$$\alpha_{\phi} = \beta_{\phi} \tag{88}$$

can be easily obtained by comparing the job creation condition in the decentralized equilibrium and the planner's equilibrium condition, allowing the bargaining power and the matching function elasticity to vary by location-skill groups. The second condition is obtained by equating the spatial equilibrium condition of the decentralized equilibrium and the spatial optimal condition of the planner

$$b^{\chi} + \frac{\alpha}{1-\alpha} k^{\chi}\theta_j^{\chi} = \left(b^{\chi} + \frac{\beta}{1-\beta} k^{\chi}\theta_j^{\chi}\right) R_j^{-\eta}$$

Re-arrange the equation and express it in terms of $\beta_j^{\chi}$, the equation becomes

$$\beta_j^{\chi} = 1 - \left[1 + \frac{R_j^{\eta}\left(b^{\chi} + \frac{\alpha}{1-\alpha} k^{\chi}\theta_j^{\chi}\right) - b^{\chi}}{k^{\chi}\theta_j^{\chi}}\right]^{-1} \tag{89}$$

Nevertheless, equation (88) and equation (89) can only be simultaneously satisfied if the decentralized spatial equilibrium condition becomes

$$\overline{U}^{\chi} = \left(b^{\chi} + \frac{\beta}{1-\beta} k^{\chi}\theta_j^{\chi}\right)$$

which happens when $R_j^{-\eta} = R_{j'}^{-\eta}$ since $R_j^{-\eta}$ will be dropped out of the spatial equilibrium condition. There are two possibilities for $R_j^{-\eta} = R_{j'}^{-\eta}$ to hold, we need either $\eta = 0$ or $R_j = R_{j'}$. Therefore, equation (88) and equation (89) can only be simultaneously satisfied either $\eta = 0$ or $R_j = R_{j'}$ holds.

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